Math 20A: Calculus for Science and Engineering I
Midterm Exam 2

Winter 2022

- You will have **50 minutes** to complete this exam.
- Please have your student ID easily accessible to show to a proctor when asked.
- You may use one 8.5 x 11 inch sheet of handwritten notes, but no calculators, phones, or other study aids.
- Simplify all answers, unless specified otherwise. (**Note: it is okay for your answers to contain fractions in lowest terms, as well as numbers like e and π and $\sqrt{2}$.**)
- Please show your work and explain your answers for each problem unless otherwise specified– we will not award full credit for the correct numerical answer without proper explanation.
- Please write your final answer for each problem in the indicated area. If you do any work on the backs of the pages or on additional scratch paper that you would like to have graded, **please indicate that clearly; otherwise it will not be graded**.
- Don't forget to write your name on the top of every page.
- Good luck!

Name: Solutions

PID:________________________

Seat Number:____________

Name:

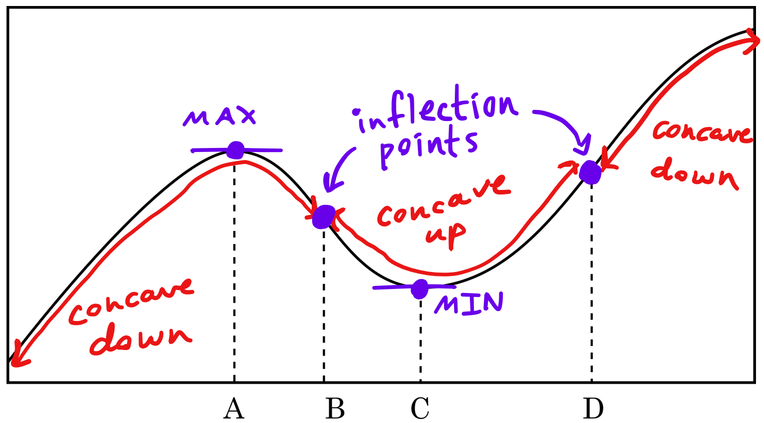
Problem 1: (8 points) For each curve property below, identify all the labeled points on the graph having that property.
(You do not need to show your work or justify your answers for this problem.)

horizontal tangent

Property	Point(s)
$f' = 0$	A, C
$f'' = 0$	B, D
$f'' < 0$	A
$f'' > 0$	C

concave down

concave up



Problem 2: (4 points) Calculate the following derivative: $\dfrac{d}{dx}\left(e^{x^4-x}\right)$.
(You do not need to show your work or justify your answers for this problem.)

Choose one:

- ○ $\dfrac{4x^3-1}{x^4-x}$
- ○ $(x^4-x)\cdot e^{x^4-x-1}$
- ● $(4x^3-1)\cdot e^{x^4-x}$
- ○ e^{x^4-x}

Use chain rule: $f(x) = e^x$ $\quad f'(x) = e^x$

$g(x) = x^4 - x \quad g'(x) = 4x^3 - 1$

Then $f(g(x)) = e^{x^4-x}$, and

$$\frac{d}{dx}\left(e^{x^4-x}\right) = f'(g(x))\cdot g'(x) = \boxed{e^{x^4-x}\cdot(4x^3-1)}$$

Problem 3: (4 points) For the function $f(x) = x^2 2^x$, there is at least one value c in $(0,1)$ such that:
(You do not need to show your work or justify your answers for this problem.)

Choose one:

- ○ $f'(c) = -2$
- ○ $f'(c) = 0$
- ● $f'(c) = 2$
- ○ $f'(c)$ DNE
- ○ Not enough information.

The Mean Value Theorem states that if $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b), then there is at least one value c in (a,b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.

Since $x^2 2^x$ is continuous and differentiable everywhere, we can apply the MVT to say that

$$f'(c) = \frac{(1)^2 2^1 - (0)^2 2^0}{1-0} = \frac{2-0}{1-0} = \boxed{2}$$ for at least one value of c in $(0,1)$.

Name:

Problem 4: (10 points) Compute the following limit: $\lim_{x\to\infty} \frac{\ln(x)}{x}$.

First, notice that the limit has the indeterminate form $\frac{\infty}{\infty}$. So we apply L'Hôpital's Rule:

$$\lim_{x\to\infty} \frac{\ln(x)}{x} = \lim_{x\to\infty} \frac{\frac{d}{dx}\ln(x)}{\frac{d}{dx}(x)}$$

$$= \lim_{x\to\infty} \frac{1/x}{1}$$

$$= \lim_{x\to\infty} \frac{1}{x}$$

as x gets large, 1/x gets small

$$= \boxed{0}$$

Answer:

0

Problem 5: (12 points) Find the linearization of $f(x) = \sin(x)$ at $x = 0$, and then use it to approximate $\sin(.001)$ *(all angles are taken in radians)*.

We know,

$f(x) = \sin x$

$f'(x) = \frac{d \sin x}{dx} = \cos x$. Then $f'(0) = 1$

Also, we know that, $f(0) = \sin(0) = 0$

Then, the linearization at $x=0$ for $f(x)$ is the equation of the tangent drawn at $x=0$ of the function $f(x)$.

∴ Equation of the tangent is;

$y = f(x) + f'(x)(x - x_1)$

$y = f(0) + f'(0)(x-0)$

$y = 0 + 1(x-0)$ $[f'(0) = 1,\ f(0) = 0]$

$y = 1(x)$ $\longrightarrow$ Linearization

However, Now to Approximate $\sin(0.001)$, we plug in 0.001 into equation above to get,

$y = 0.001 \approx \sin(0.001)$

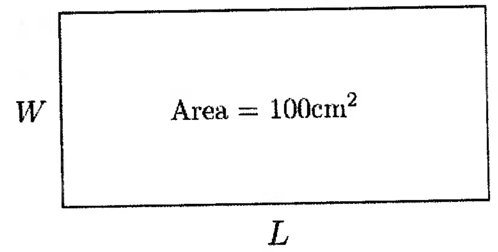
Linearization:

$y = x$

Approximation of sin(.001):

0.001

Problem 6: (12 points) A rectangle has a constant area of 100 cm^2, and its width W is decreasing at a constant rate of 0.5 cm/sec. At the moment when its width is 5 cm, how quickly is its length L increasing?



$100 \text{ cm}^2 = W \cdot L$

want : $\frac{dL}{dt}$

$\frac{d}{dt}$

know : $\frac{dW}{dt} = -0.5$ cm/sec

when $W = 5$ cm

$\frac{d}{dt}(100) = \frac{d}{dt}(W \cdot L)$

$0 = \frac{dW}{dt} \cdot L + W \cdot \frac{dL}{dt}$

$-\frac{dW}{dt} \cdot L = W \cdot \frac{dL}{dt}$

$\frac{-\frac{dW}{dt} \cdot L}{W} = \frac{dL}{dt}$

Variables

$W = W(t)$ width of rectangle

$L = L(t)$ length of rectangle

when $W = 5$

$100 = 5 \cdot L$

$20 = L$

$\frac{dL}{dt} = \frac{-(-0.5)\cdot 20}{5}$

$= \frac{10}{5}$

$\frac{dL}{dt} = 2$

Answer:

$\boxed{2 \text{ cm/sec}}$

Problem 7: (3 points) _Bonus problem_ – *I recommend focusing on the other problems and trying this one only if you have extra time!*

Find a point at which the curve $x^2 + y^2 = 2xy + 2y$ has a horizontal tangent line.

$$x^2 + y^2 = 2xy + 2y$$

$$2x + 2y \cdot y' = 2y + 2xy' + 2 \cdot y'$$

$$2x - 2y = 2xy' + 2y' - 2yy'$$

$$2x - 2y = y'(2x + 2 - 2y)$$

$$y' = \frac{2x - 2y}{2x + 2 - 2y}$$

horizontal tangent line means slope $= 0$

$$\frac{2x - 2y}{2x + 2 - 2y} = 0$$

$$2x - 2y = 0$$

$$2x = 2y$$

$$x = y$$

sub into original equation:

$$x^2 + (x)^2 = 2x(x) + 2(x)$$

$$2x^2 = 2x^2 + 2x$$

$$2x = 0$$

$$x = 0$$

and if $x = y$

then $y = 0$

so the point is $(0,0)$ //

Answer:

$(0,0)$