

# Math 20A: Calculus for Science and Engineering I

## Midterm Exam 1

Winter 2022

- You will have **50 minutes** to complete this exam.
- Please have your student ID easily accessible to show to a proctor when asked.
- You may use one 8.5 x 11 inch sheet of handwritten notes, but no calculators, phones, or other study aids.
- Simplify all answers, unless specified otherwise. (**Note: it is okay for your answers to contain fractions in lowest terms, as well as numbers like  $e$  and  $\pi$  and  $\sqrt{2}$ .**)
- Please show your work and explain your answers for each problem unless otherwise specified—we will not award full credit for the correct numerical answer without proper explanation.
- Please write your final answer for each problem in the indicated area. If you do any work on the backs of the pages or on additional scratch paper that you would like to have graded, **please indicate that clearly; otherwise it will not be graded.**
- Don't forget to write your name on the top of every page.
- Good luck!

Name: \_\_\_\_\_

PID: \_\_\_\_\_

Seat Number: \_\_\_\_\_

Name: \_\_\_\_\_

**Problem 1: (4 points)** The following limit represents a derivative  $f'(a)$ . Identify  $f(x)$  and  $a$ .  
(You do not need to show your work or justify your answers for this problem.)

$$\lim_{h \rightarrow 0} \frac{\cos(\frac{\pi}{3} + h) - \frac{1}{2}}{h}$$

$f(x) =$  \_\_\_\_\_       $a =$  \_\_\_\_\_

**Problem 2: (5 points)** Which of the following is an equation of the line tangent to the graph of  $y = x + e^x$  at  $x = 0$ ?

(You do not need to show your work or justify your answers for this problem.)

Choose one:

- $y = x$
- $y = x + 1$
- $y = x + 2$
- $y = 2x$
- $y = 2x + 1$

**Problem 3: (5 points)** What is the slope of the curve  $y = 10x^2 - 3x - 1$  at the point where it crosses the positive part of the  $x$ -axis?

(You do not need to show your work or justify your answers for this problem. Also, note/clarification: "slope of the curve" is another way of saying "slope of the line tangent to the curve.")

Choose one:

- 3/20
- 1/5
- 1/3
- 7

Name: \_\_\_\_\_

**Problem 4: (11 points)** Let  $f(x) = (1 + x)e^x$ .

(a) (5 points) Find  $f'(x)$ . (You may use any rules you know.)

Answer:

(b) (6 points) Find  $f^{(3)}(-1)$ . (You may use any rules you know.)

Answer:

Name: \_\_\_\_\_

**Problem 5: (15 points)** Evaluate the following limits if they exist. (If the limit is  $\infty$  or  $-\infty$ , say so. Do not use L'Hôpital's rule.)

(a) (5 points)  $\lim_{x \rightarrow 1} \sqrt[3]{7 + x^5}$

Answer:

(b) (5 points)  $\lim_{t \rightarrow -1} \frac{\sqrt{2t + 5} - \sqrt{3}}{t + 1}$

Answer:

(c) (5 points)  $\lim_{\theta \rightarrow 0} \frac{\sin(5\theta)}{\sin(8\theta)}$

Answer:

Name: \_\_\_\_\_

**Problem 6: (10 points)** Let  $f(x)$  be a function satisfying

$$8 - x^2 \leq f(x) \leq 8 + x^2$$

for all values of  $x$  in the interval  $(-\frac{1}{2}, \frac{1}{2})$ .

(a) (7 points) Find the limit  $\lim_{x \rightarrow 0} f(x)$  (or state that there is not enough information to find it). Be sure to show your work and justify your reasoning.

*(If you are going to use a theorem, name the theorem and verify that any hypotheses are satisfied.)*

Answer:

Name: \_\_\_\_\_

**Problem 6 (continued)**

(b) (3 points) Is  $f$  continuous at  $x = 0$ ?

(Answer "yes," "no," or "not enough information," and justify your answer.)

Answer: