Announcements

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- Midterm 1 in person Mon, Jan 31.
- Check email for details (seating, practice test, etc.)
- Recorded lecture this Wed (I have jury duty)
- Monday SI time moved to Fridays

Today finish §3.3: Product and Quotient Rules
\+ §3.5: Higher Derivatives
\+ start §3.4: Rates of Change

Last Time **Product Law:** If f and g are differentiable functions, then fg is differentiable, and

$$\frac{d}{dx}(fg) = \frac{df}{dx}\cdot g + f\cdot\frac{dg}{dx} \qquad \left((fg)' = f'g + fg' \right)$$

Warm-up problem Find $\frac{d}{dx} e^{2x}$.
(Hint: can you use the product rule?)

Try it if you have time!

Notice $e^{2x} = e^x \cdot e^x$

$$(e^{2x})' \overset{\text{PROD RULE}}{=} (e^x)' e^x + e^x (e^x)'$$

$$= e^x \cdot e^x + e^x \cdot e^x$$

$$= \boxed{2 \cdot e^{2x}}$$

Quotient Rule Given differentiable functions f and g, then $\frac{f}{g}$ is differentiable at all points x for which $g(x) \neq 0$, and

$$\left(\frac{f}{g}\right)' = \frac{g\cdot f' - fg'}{g^2} \qquad \frac{d}{dx}\frac{f}{g} = \frac{g\cdot\frac{df}{dx} - f\cdot\frac{dg}{dx}}{g^2}$$

Can prove it using the product rule: $\frac{f}{g} = f \cdot \frac{1}{g}$

Mnemonic: $D\left(\frac{Hi}{Lo}\right) = \frac{Lo\; d\,Hi - Hi\; d\,Lo}{Lo^2}$

Ex $h(x) = \frac{x^2+2}{x}$. Find $h'(x)$.

$$h'(x) = \frac{x\,\frac{d}{dx}(x^2+2) - (x^2+2)\frac{d}{dx}x}{x^2} = \frac{x(2x+0)-(x^2+2)\cdot 1}{x^2}$$

$$= \frac{2x^2-(x^2+2)}{x^2} = \boxed{\frac{x^2-2}{x^2}}$$

Ex $g(x) = e^{-x}$. Find $g'(x)$.

Notice: $g(x) = \frac{1}{e^x}$.

$$g'(x) = \frac{\underbrace{\cancel{e^x\cdot\frac{d}{dx}1}}_{0} - 1\cdot\underbrace{\frac{d}{dx}e^x}_{e^x}}{(e^x)^2}$$

$$= \frac{\boxed{-\cancel{e^x}}}{(e^x)^{\cancel{2}}} \leftarrow = -1\cdot e^x$$

$$= -\frac{1}{e^x} = -e^{-x}$$

§3.5 "Higher derivatives" are obtained by repeatedly differentiating a function $f(x)$. If $f'(x)$ is differentiable, then

$$f''(x) = \frac{d}{dx}f'(x)$$ "second derivative" of f

Ex $f(x) = x^2 + \underbrace{\sqrt{x}}_{x^{1/2}} \longrightarrow f'(x) = 2x^1 + \frac{1}{2}x^{-1/2}$

$$f''(x) = \frac{d}{dx}f'(x) = 2 + \frac{1}{2}\cdot\left(-\frac{1}{2}\right)\cdot x^{-3/2}$$

$= 2 - \frac{\;}{4}x$ could go on...

Notation function $f(x)$

First	Second	Third	...	"n^{th}"
$f'(x)$	$f''(x)$	$f'''(x)$		confusing ↓ instead
$\frac{df}{dx}$	$\frac{d^2f}{dx^2}$	$\frac{d^3f}{dx^3}$		$\frac{d^nf}{dx^n}$
$f^{(1)}(x)$	$f^{(2)}(x)$	$f^{(3)}(x)$		$f^{(n)}(x)$

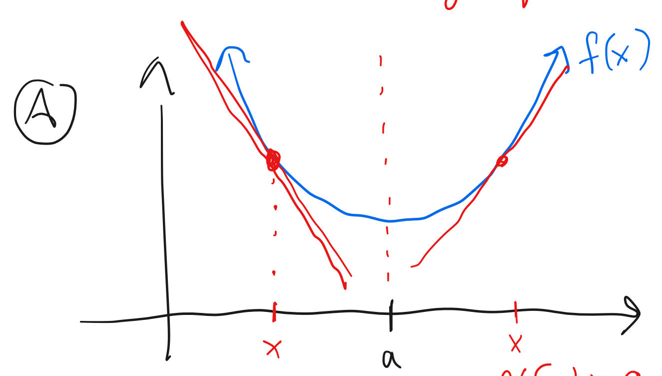
<u>Warning</u> $f^{(3)}(x)$ is <u>not</u> $[f(x)]^3$ (i.e. not exponent)

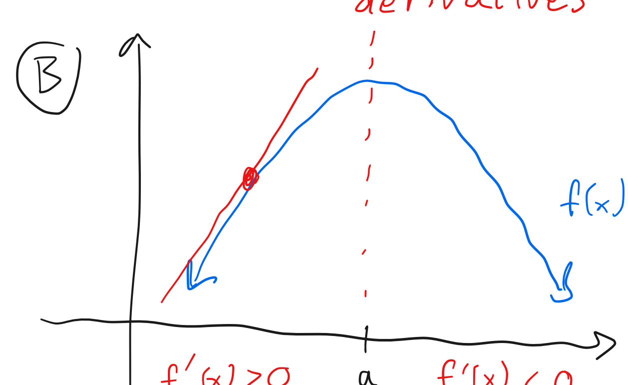
<u>Note</u> $\frac{df}{dx} = \frac{d}{dx} f \longrightarrow \left(\frac{d^3}{dx^3}\right)(f) = \frac{d^3f}{dx^3}$

<u>Ex</u>
$$f(x) = 2x^3 + 7x + 1 \qquad \text{degree 3 polynomial}$$
$$f'(x) = 6x^2 + 7 + 0$$
$$f''(x) = 12x^1 + 0$$
$$f'''(x) = 12 + 0$$
$$f^{(4)}(x) = 0$$
$$\vdots$$
$$f^{(n)}(x) = 0 \qquad \text{for } n \geq 5$$

<u>Q</u> What do derivatives tell us about graphs?

graph derivatives





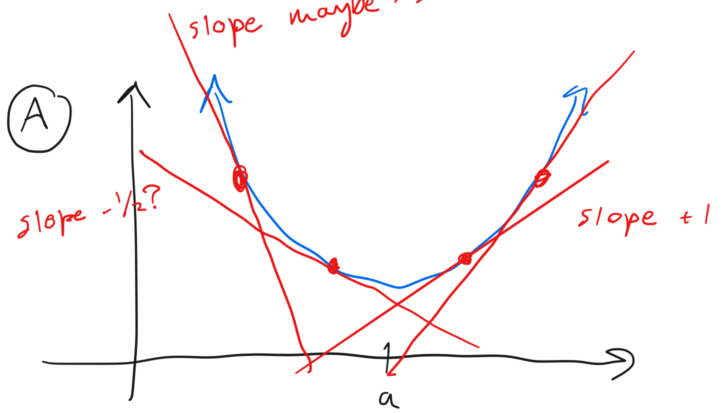
$f'(x) < 0$
f is decreasing
for $x < a$

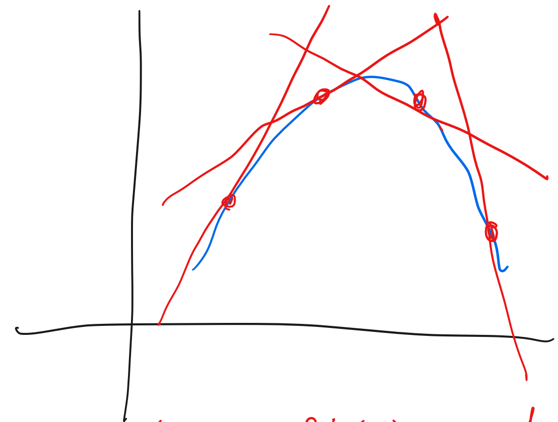
$f'(x) > 0$
$f(x)$ is increasing
for $x > a$

f increasing
$x < a$

$f'(x) < 0$
f decreasing
$x > a$

- $f'(x) > 0$ means f increasing, $f'(x) < 0$ means f decreasing
- $f''(x) > 0$ $\qquad f''(x) < 0$??



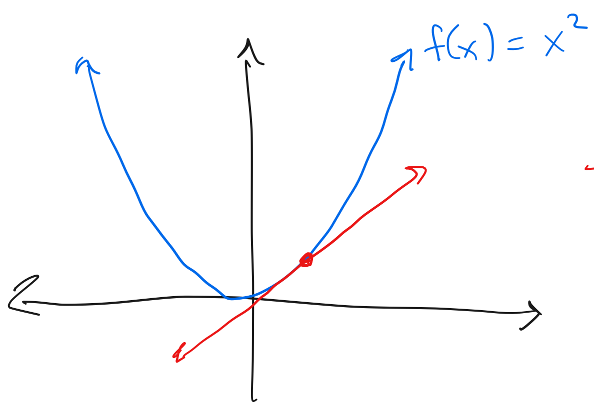


Here slopes $f'(x)$ are increasing
The derivative of $f'(x)$ is > 0
I.e $f''(x) > 0$

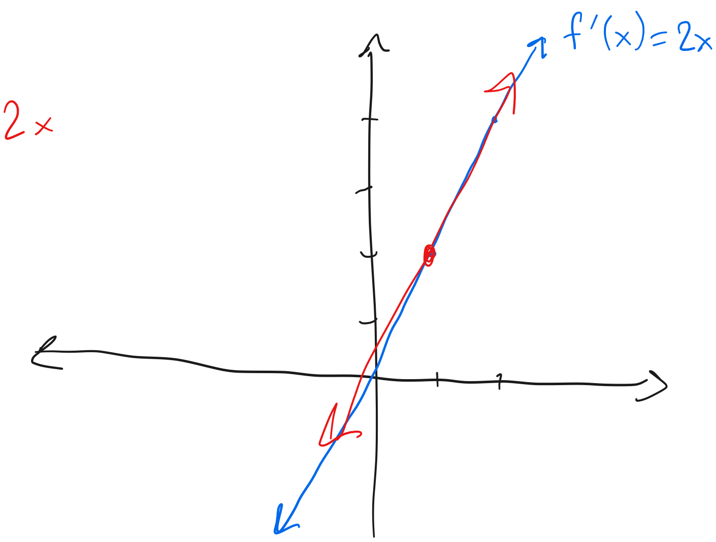
slopes $f'(x)$ are decreasing
$f''(x) < 0$

- $f'' > 0$ means f is concave up
- $f'' < 0$ means f is concave down

student question after class



$f'(x) = 2x$



$f''(x) = 2$