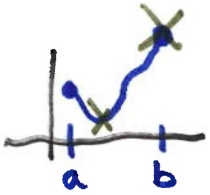
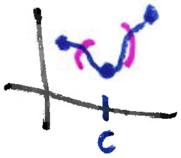
Recap of last time:



Theorem 1. *(Extreme Value Theorem)*
A continuous function f on a closed interval $I = [a, b]$ attains both a maximum and a minimum value on I.



Definition We say that $f(c)$ is a local minimum at $x = c$ if $f(c)$ is the minimum value of f on some open interval I containing c (I in the domain of f).

Theorem 2. *(Fermat's Theorem on Local Extrema)*
If $f(c)$ is a local min or max, then c is a "critical point" of f (this means $f'(c) = 0$ or $f'(c)$ DNE).

Theorem 3. *(Extreme Values on a Closed Interval)*
A continuous function f on a closed interval $I = [a, b]$ attains its absolute max and absolute min, and each of these occurs at a critical point or one of the endpoints.

Problem *(Try it if you have time!)*
Find the extreme values of $f(x) = x^2 - 8\ln(x)$ on $[1, 4]$.

Step 1 f is continuous on [1,4] ✓

Step 2 Find critical points:

$$f'(x) = 2x - 8 \cdot \frac{1}{x}$$

Have $f'(x) = 0$ if $2x = \frac{8}{x} \Rightarrow 2x^2 = 8$

$x^2 = 4$

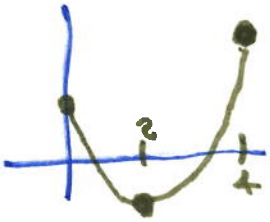
$x = \pm 2 \rightarrow \boxed{x=2}$

only critical pt. in [1,4]

Plug in: $f(2) = 2^2 - 8\ln(2) \approx -1.5$

Step 3 endpoints: $f(1) = 1^2 - 8\overset{0}{\ln(1)} = 1$

$f(4) = 4^2 - 8\ln(4) \approx 4.9$



max $f(4) \approx 4.9$

min $f(2) \approx -1.5$