

# Math 180A: Introduction to Probability

## Quiz 4

Fall 2021

- You will have **50 minutes** to complete this quiz.
- Please have your student ID easily accessible to show to a proctor when asked.
- You may use one 8.5 x 11 inch sheet of handwritten notes, but no calculators, phones, or other study aids are permitted.
- Unless stated otherwise, if a question calls for a numerical answer, you don't need to simplify. **(For example, it's okay to write something like  $(0.9 - 0.01)7!/\binom{3}{2}$  as your answer.)**
- Please show your work and explain your answers for each problem unless otherwise specified—we will not award full credit for the correct numerical answer without proper explanation.
- Please write your final answer for each problem in the indicated area. If you do any work on the backs of the pages or on additional scratch paper that you would like to have graded, **please indicate that clearly; otherwise it will not be graded.**
- Don't forget to write your name on the top of every page.
- Good luck!

Name: \_\_\_\_\_

PID: \_\_\_\_\_

Seat Number: \_\_\_\_\_

Name: \_\_\_\_\_

**Problem 1: (1 points)** You flip 3 fair coins independently. Let  $X$  be the number of heads observed and let  $Y$  be the number of tails. Are  $X$  and  $Y$  independent random variables?  
(You do not need to show your work or justify your answers for this problem.)

Choose one:

- Yes
- No

**Problem 2: (1 points)** If  $X$  is an exponential random variable with parameter 2, which of the following is equal to the conditional probability  $P(X > 10|X > 9)$ ?  
(You do not need to show your work or justify your answers for this problem.)

Choose one:

- $1 - e^{-2 \cdot 1}$
- $P(X > 1)$
- $P(X > 9)$
- $e^{-2 \cdot 10}$

**Problem 3: (2 points)** Let  $X$  be a normal random variable with mean 1 and variance 4. Which of the following is equal to  $P(|X - 1| \geq 2)$ ?  
(You do not need to show your work or justify your answers for this problem.)

Choose one:

- $\Phi(0) + (1 - \Phi(2))$
- $\Phi(1) + \Phi(-1)$
- $2 \cdot (1 - \Phi(1))$
- $\Phi(1/2)$

Name: \_\_\_\_\_

**Problem 4: (10 points)** Let  $X$  and  $Y$  be continuous random variables with joint distribution

$$f_{X,Y}(x,y) = \frac{6}{7} \cdot y, \quad \text{for } x \in (0,1) \text{ and } y \in (0, x+1)$$

*You may express your answers for parts (a) through (c) in terms of explicit but unevaluated integrals.*

(a) (3 points) Compute  $F_{X+Y}(1)$ .

Answer:

Name: \_\_\_\_\_

**Problem 4 (continued)**

(b) (3 points) Compute  $P(X \leq \frac{1}{2})$ .

Answer:

Name: \_\_\_\_\_

**Problem 4 (continued)**

(c) (2 points) Find the marginal PDF of  $X$ .

Answer:

(d) (2 points) Are  $X$  and  $Y$  independent? Why or why not?

Answer:

Name: \_\_\_\_\_

**Problem 5: (10 points)** Let  $X$ ,  $Y$ , and  $Z$  be independent exponential random variables with parameter 2.

*You may express your answers for this problem in terms of explicit but unevaluated integrals; however, your answer for part (b) should not contain any unevaluated “max” terms.*

(a) (2 points) Find  $E(X^2Y)$ .

Answer:

(b) (4 points) Find the probability  $P(\max(X, Y) \leq Z)$ .

Answer:

Name: \_\_\_\_\_

**Problem 5 (continued)**

(c) (4 points) Find the probability density function of  $X - Y$ .

Answer: