Math 180A Quiz 1 Solutions

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October 9, 2021

1. If A and B are disjoint events, which of the following are always true?

   - ☑ $P(A|B) = 0$
   - ☐ $B = A^c$
   - ☑ $P(A \cup B) = P(A) + P(B)$
   - ☐ $P(A \cap B) = P(A) \cdot P(B)$

   Solution: As A and B are disjoint events, $P(A \cap B) = 0$. Thus $P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B)$ which allows us to pick the third answer. We also have that $P(A|B) = \frac{P(A\cap B)}{P(B)}$, so the first answer is also valid for every A, B for which $P(A|B)$ is defined. Thus we pick the first and third answers. For the second answer, we can find disjoint events A, B such as the event that a 6-sided die rolls a 1 and the event that the same die rolls a 5, which are certainly disjoint but $B \neq A^c$. Finally, as we know that $P(A \cap B) = 0$, the same disjoint events show that $P(A \cap B) \neq P(A)P(B)$

2. Suppose A, B and C are events with

   $$P(A) = P(B) = P(C) = .3$$

   and

   $$P(A \cap B) = P(A \cap C) = P(B \cap C) = .1$$

   What is the value of $P(A \cup B \cup C)$?

   - ☐ .6
   - ☐ .7
   - ☐ .8
   - ☑ Not enough information

   Solution: Using inclusion-exclusion, we can write

   $$P[A \cup B \cup C] = P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$$

   We have values for the the first six terms, but we do not have any value for $P(A \cap B \cap C)$. Since $P(A \cap B \cap C)$ cannot be determined by the values of of the other events, and we have no other information, we cannot give a value for $P(A \cap B \cap C)$. So we do not have enough information.

3. Of the customers ordering burgers at In-N-Out, suppose that **30%** ask for their burger "animal style," **15%** ask for their burger with chopped chilis, and **10%** ask for both (i.e. they ask for their burger "animal style with chopped chilis").

(a) What is the probability that a randomly chosen customer orders their burger neither animal style nor with chopped chilis?

$P(A)$ = ask for "animal style" = 0.3

$P(B)$ = ask for chopped chilis = 0.15

$P(A \cap B)$ = both = 0.1

Want to find $P(A^c \cap B^c) = P(A \cup B)^c$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.15 - 0.1$

$= 0.35$

$P(A \cup B)^c = 1 - P(A \cup B) = 1 - 0.35 = 0.65$

Answer:

0.65

(b) Given that a customer orders their burger with chopped chilis, what is the conditional probability that they also ask for it "animal style"?

$P(\text{animal style} \mid \text{chopped chilis}) = \frac{P(\text{animal} \cap \text{chilis})}{P(\text{chilis})}$

$= \frac{.1}{.15}$

Answer:

$\frac{.1}{.15}$

4. You and your friend each choose a number between 1 and 10 uniformly at random (you choose the numbers without consulting each other). We define the following events:

$$A = \{\text{your number is equal to your friend's number}\}$$

$$B = \{\text{the sum of your number and your friend's number is } 4\}$$

(a) Give a sample space Ω and a probability measure P for this experiment.

$\boxed{\Omega = \{ (x,y) \in \mathbb{Z}^2 : x,y \in [1,10] \}}$

Order matters in the sense that you choosing a number and your friend choosing that number are different outcomes, e.g. $(1,2) \neq (2,1)$, but for events A and B they are symmetric.

$P(x) = \frac{|X|}{10 \cdot 10} = \frac{|x|}{100}$ because uniform

Sample space:

$\Omega = \{ (x,y) \in \mathbb{Z}^2 : x,y \in [1,10] \}$

Probability measure:

$X \subseteq \Omega$, then $P(X) = \frac{|X|}{100}$ because uniform

For any $(a,b) \in \Omega$, $P((a,b)) = \frac{1}{100}$

(b) What is $P(A)$?

(3 points) What is $P(A)$?

$P(A) = \frac{|A|}{100} = \frac{10}{100} = \frac{1}{100}$

$A = \{(1,1), \ldots, (10,10)\}$
there are 10 events
in this subset

Answer:

$P(A) = \frac{1}{10}$

(c) What is $P(A|B)$?

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{(1/100)}{(3/100)}$

$P(B) = \frac{3}{10 \cdot 10}$ (1, 3) (3 1)
(2, 2)

$P(A \cap B)$ = Same number, adds up to 4: (2, 2)

Answer:

1/3

5. Suppose that among the students in Math 180A, there are:

- 50 Sophomores
- 70 Juniors
- 30 Seniors

A committee of 10 students is chosen uniformly at random from among the students in the class.

(a) What is the probability that exactly 5 sophomores, 3 juniors, and 2 seniors are chosen?

$$\frac{\binom{50}{5}\cdot\binom{70}{3}\cdot\binom{30}{2}}{\binom{150}{10}}$$

← total ways to choose 10 students from a class of 150 (#Ω)

$\binom{50}{5}$ = all the ways to choose 5 sophomores from 50

$\binom{70}{3}$ = " - - - - - to choose 3 juniors from 70 "

$\binom{30}{2}$ = " - - - - - to choose 2 seniors from 30 "

Answer:

$$\frac{\binom{50}{5}\cdot\binom{70}{3}\cdot\binom{30}{2}}{\binom{150}{10}}$$

(b) What is the probability that at least one sophomore is chosen?

Think complement. No sophomore is chosen.

Let S be the event in which no sophomore is chosen.

Then $P(S) = \binom{70+30}{10}$ choosing 10 amongst juniors and seniors.

$$= \frac{\binom{100}{10}}{\binom{150}{10}}$$

We want the complement

$P(S^c) = 1 - P(S)$

$$= 1 - \frac{\binom{100}{10}}{\binom{150}{10}}$$ ← # sample space

Answer:

$$1 - \frac{\binom{100}{10}}{\binom{150}{10}}$$