Announcements:

- HW 1 due tonight - 11:59 PM
- 2 OHs after class - Jiaqi Liu 5:00 pm - 6:30 pm
  Pooja Agarwal 6:30 pm - 8:30 pm
- Quiz 1 tomorrow / Wednesday
  ↳ Gradescope

Quiz 1 Review

① You flip a coin and roll a 6-sided fair die. Describe the sample space of this experiment

probability

$\Omega = \{(a,i) : a \in \{H,T\} \quad 1 \le i \le 6\} \quad i \in \mathbb{Z}$

$\#\Omega = 2 \times 6 = 12$

For any $\omega \in \Omega$ $\mathbb{P}(\{\omega\}) = \frac{1}{\#\Omega} = \frac{1}{12}$

What is the probability that the flip is head or the roll is a 3?

$A = \{\text{Flip is head}\}$ $B = \{\text{Roll is 3}\}$ → Not disjoint

$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(AB)$

$= \frac{1}{2} + \frac{1}{6} - \frac{1}{12} = \frac{7}{12}$

$AB = (\text{Flip is head}, \text{Roll is 3})$

$= (H,3)$

$\mathbb{P}(AB) = \frac{1}{12}$

We roll a 6-sided unfair die; where even numbers are twice as likely as the odd numbers.

$\Omega = \{1, 2, 3, 4, 5, 6\}$

$\mathbb{P}(\{\omega\}) \neq \frac{1}{6} \quad \forall \omega \in \Omega$

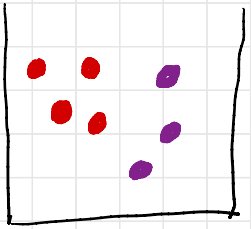
$\mathbb{P}(\{\omega\}) = \frac{2}{9}$ if ω is even $\quad \mathbb{P}(\{\omega\}) = \frac{1}{9}$ if ω is odd

Solution

$\mathbb{P}(\{\omega\}) = x$ if ω is odd $\quad \mathbb{P}(\{\omega\}) = 2x$ if ω is even

$\mathbb{P}(\Omega) = 1 = x(3) + 2x(3) \Rightarrow x = 1/9$

② A bin contains 4 red balls and 3 purple balls.



- You pick 2 balls with replacement.
  What is the probability you picked at least 1 red ball.

With replacement — Ordered

$\Omega = \{(b_1, b_2) : 1 \le b_1, b_2 \le 7\}$ $\quad \#\Omega = 7 \times 7 = 49$

$A = \{$ at least one red ball $\}$

exactly 1R or exactly 2R

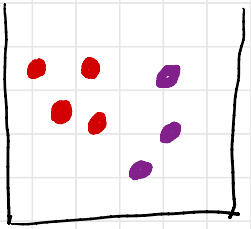
$A^c = \{$ no red ball $\}$

$\#A^c = 3 \times 3 = 9$

$$P(A) = 1 - \frac{9}{49} = \frac{40}{49}$$

$$P(A^c) = \frac{9}{49}$$

(3) A bin contains 4 red balls and 3 purple balls.



- You pick 2 balls without replacement.

What is the probability you picked at least 1 red ball or at least 1 purple ball

Without Replacement Without order

$\Omega = \{(b_1, b_2) : b_i \neq b_j \text{ if } i \neq j\}$

$A = \{\text{at least 1 red}\}$ $B = \{\text{at least 1 purple}\}$ $\#\Omega = \binom{7}{2}$

$P(A \cup B)$

$(A \cup B)^c$ = no red, no purple

$P((A \cup B)^c) = 0$

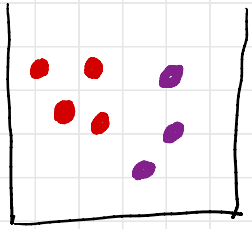
$= 1$

$A^c = \{\text{no red ball}\}$

$\#A^c = \binom{3}{2} = 3$

$P(A^c) = \dfrac{\binom{3}{2}}{\binom{7}{2}}$

④ A bin contains 4 red balls and 3 purple balls.



- You pick 2 balls without replacement. What is the probability the second ball is red?

(5) You choose a number between 1 and 20.
What is the probability that it is divisible by 3?

What is the probability that it is divisible by 3 given it is even?

⑥ You pick 3 cards from a deck of 52 without replacement. What is the probability that you picked at least 2 queens?

Next – 2.1 Total probability

2.2 Bayes' formula