Math 180A Homework 7

Fall 2021

Due date: **11:59pm** (Pacific Time) on **Mon., Nov. 15** (via Gradescope)

In the "collaborators" field in Gradescope, please write a list of everyone with whom you collaborated on this assignment, as well as any outside sources you consulted, apart from the textbook and your notes. If you did not collaborate with anyone, please explicitly write, "No collaborators."

Section 1 (input directly in Gradescope)

Submit the answers to these problems directly through the Gradescope interface. You do not need to write up or explain your work.

Problem 1 (numerical answers)**.** The joint probability mass function of the random variables (X, Y) is given by the following table:

$X \backslash Y$	0	1	2	3
1	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{15}$
2	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{10}$
3	$\frac{1}{30}$	$\frac{1}{30}$	0	$\frac{1}{10}$

(a) Calculate the probability $P(Y - X \geq 1)$.

(b) Calculate the probability $P(X + Y^2 \leq 2)$.

Section 2 (upload files)

For each problem, write your solution on a page by itself, and upload it as a separate file to Gradescope (either typed or scanned from handwritten work). You should write your solutions to these problems neatly and carefully and provide full justification for your answers.

Problem 2 (ASV Exercise 6.5)**.** Suppose X, Y have joint density function

$$f(x,y) = \begin{cases} \frac{12}{7}(xy + y^2), & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Check that f is a genuine joint density function.

(b) Find the marginal density functions of X and Y.

(c) Calculate the probability $P(X < Y)$.

(d) Calculate the expectation $E[X^2 Y]$.

Problem 3 (ASV Exercise 6.29)**.** Suppose that $X \sim \text{Geom}(p)$ and $Y \sim \text{Geom}(r)$ are independent. Find the probability $P(X < Y)$.

Problem 4 (ASV Exercise 6.36)**.** Suppose that X, Y are jointly continuous with joint probability density function

$$f(x, y) = ce^{-\frac{x^2}{2} - \frac{(x-y)^2}{2}}, \quad (x, y) \in \mathbb{R}^2,$$

for some constant c.

(a) Find the value of the constant c.

(b) Find the marginal density functions of X and Y.

(c) Determine whether X and Y are independent.

Problem 5. You have a lamp with two different lightbulbs of different types. The lifetime of the first lightbulb (in years) is described by an exponential random variable X with mean 1, and the second lightbulb's lifetime is described by an exponential random variable Y with mean 2, independent from X. Let T be the amount of time from now until either one of the lightbulbs burns out, so $T = \min(X, Y)$.

(a) Find the cumulative distribution function of T.

(b) Find the probability density function of T.

(c) Find the expectation of T.

Problem 6. John works in a tax preparation office. Each tax return that he has to complete takes a random amount of time (measured in hours) distributed uniformly on the interval $[1, 3]$, and the times for different tax returns are independent. If he finishes four tax returns before the end of his eight-hour workday, he can go home early. What is the probability that he finishes four tax returns within eight hours?

Problem 7 (ASV Exercise 7.2)**.** Let X and Y be independent Bernoulli random variables with parameters p and r, respectively. Find the distribution of $X + Y$.

Problem 8 (ASV Exercise 7.20)**.** Let X have density $f_X(x) = 2x$ for $0 < x < 1$ and let Y be uniform on the interval $(1, 2)$. Assume X and Y are independent.

(a) Give the joint density function of (X, Y).

(b) Calculate $P(Y - X \geq \frac{3}{2})$.

(c) Find the density function of $X + Y$.