Math 180A Homework 5

Fall 2021

Due date: **11:59pm** (Pacific Time) on **Mon. Nov, 1** (via Gradescope)

In the "collaborators" field in Gradescope, please write a list of everyone with whom you collaborated on this assignment, as well as any outside sources you consulted, apart from the textbook and your notes. If you did not collaborate with anyone, please explicitly write, "No collaborators."

Section 1 (input directly in Gradescope)

Submit the answers to these problems directly through the Gradescope interface. You do not need to write up or explain your work.

Problem 1 (multiple choice)**.** Which of the following would be modeled best by a Poisson random variable?

(a) The amount of time you have to wait before a car drives by your house.

(b) The number of cars that drive by your house in one hour.

(c) The number of cars that drive by your house before you see a red car.

Problem 2 (multiple choice)**.** X is a random variable, and its cumulative distribution function F_X is

$$F_X(t) = \begin{cases} 0, & t < -3 \\ 0.2, & -3 \leq t < -1 \\ 0.7, & -1 \leq t < 5 \\ 1.0, & t \geq 5. \end{cases}$$

The random variable X is

(a) Discrete

(b) Continuous.

Problem 3 (numerical answer)**.** A random variable X has probability density function

$$f(x) = \begin{cases} cx^3, & 0 \leq x \leq 1 \\ 0, & \text{otherwise}, \end{cases}$$

where c is some positive constant. What is c?

Problem 4 (numerical answer)**.** A random variable X has density function

$$f(x) = \begin{cases} 6x(1-x), & 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

Compute $E[X]$.

Section 2 (upload files)

For each problem, write your solution on a page by itself, and upload it as a separate file to Gradescope (either typed or scanned from handwritten work). You should write your solutions to these problems neatly and carefully and provide full justification for your answers.

Problem 5. Let X be a continuous random variable with probability density

$$f_X(x) = \begin{cases} 1/6, & -2 \le x \le 0, \\ 2/9, & 0 < x \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Compute $\mathbb{E}[X]$.

(b) Compute $\mathrm{Var}(X)$.

(c) Compute $\mathbb{E}[(X-1)^2]$.

Problem 6 (ASV 3.37)**.** Suppose that a random variable X has the cumulative distribution function

$$F(x) = \begin{cases} \frac{x}{1+x}, & x \ge 0 \\ 0, & x < 0. \end{cases}$$

(a) Find the probability density function f.

(b) Calculate $P(2 < X < 3)$.

(c) Calculate $E[(1+X)^2 e^{-2X}]$.

Problem 7. You and your friends go to Joshua Tree National Park on December 13 to watch the peak of the Geminid meteor shower. According to the Griffith Observatory website, the expected number of meteors you will see is 150 per hour. What is the probability that you will see at least two meteors in the first minute? **Hint:** Convert 150 meteors per hour to meteors per minute, then choose an appropriate random variable to model this situation.

Problem 8. A *standard Cauchy random variable* is a random variable X with the probability density function

$$f(x) = \frac{1}{\pi(1+x^2)}.$$

(a) Verify that f is actually a density function (it is nonnegative and integrates to 1).

(b) Show that $E(|X|) = \infty$. **Hint:** Set up $E(|X|)$ as an integral from $-\infty$ to $+\infty$, and then break it up into two pieces: $-\infty$ to 0 and 0 to $+\infty$.

(c) (Bonus) Can you find $E[X]$?

Problem 9 (Bonus – NOT TO BE TURNED IN; ASV 3.44)**.** Consider a line through the origin in $\mathbb{R}^2$ whose angle Θ is chosen uniformly at random. Let X be the slope of the line. Show that X is a standard Cauchy random variable.