Math 180A Homework 4

Fall 2021

Due date: **11:59pm** (Pacific Time) on **Mon. Oct, 25** (via Gradescope)

In the "collaborators" field in Gradescope, please write a list of everyone with whom you collaborated on this assignment, as well as any outside sources you consulted, apart from the textbook and your notes. If you did not collaborate with anyone, please explicitly write, "No collaborators."

Section 1 (input directly in Gradescope)

Submit the answers to these problems directly through the Gradescope interface. You do not need to write up or explain your work.

Problem 1 (2 points (1/2 each), numerical answers)**.** Suppose that the random variable X has expected value $E[X] = 1$ and variance $\mathrm{Var}(X) = 3$. Compute the following quantities:

(a) $E[3X + 2]$

(b) $E[X^2]$

(c) $E[(2X + 1)^2]$

(d) $\mathrm{Var}(-2X + 7)$

Section 2 (upload files)

For each problem, write your solution on a page by itself, and upload it as a separate file to Gradescope (either typed or scanned from handwritten work). You should write your solutions to these problems neatly and carefully and provide full justification for your answers.

Problem 2. You flip a fair coin. If the flip is heads, you roll a four-sided die. If the flip is tails, you roll a six-sided die. Let X be the number that comes up on the die roll. *Note:* This problem builds on HW 2, problem 5.

(a) Find the probability mass function of X.

(b) Find the expectation of X.

(c) Find the variance of X.

Problem 3 (ASV Exercise 2.21)**.** Jane must get at least three of the four problems on the exam correct to get an A. She has been able to do 80% of the problems on old exams, so she assumes that the probability she gets any problem correct is 0.8. She also assumes that the results on different problems are independent.

(a) What is the probability she gets an A?

(b) If she gets the first problem correct, what is the probability she gets an A?

Problem 4. Ten friends go to an arcade and buy 20 game tickets each. Each ticket allows them to play one arcade game. The arcade will give a plush tanuki[1] to any person who wins 12 games.

(a) If one person plays 20 games, and wins each game with probability p (independent of the other games), what is the probability that the person wins a plush tanuki? (The answer will be a formula depending on p.)

(b) Assume each of the ten friends plays 20 games, each friend wins each game with probability p, and the outcomes of the different games are all independent. What are the chances that at least one of the ten friends wins a plush tanuki?

(c) (Bonus — NOT TO BE TURNED IN) How could you modify the assumptions to be more realistic or interesting? What methods would you use to solve your modified version of the problem?

Problem 5 (ASV Exercise 2.61)**.** Suppose an urn has 3 green balls and 4 red balls.

(a) Nine draws are made with replacement. Let X be the number of times a green ball appears. Identify by name the probability distribution of X. Find the probabilities $P(X \geq 1)$ and $P(X \leq 5)$.

(b) Draws with replacement are made until the first green ball appears. Let N be the number of draws that are needed. Identify by name the probability distribution of N. Find the probability $P(N \leq 9)$.

(c) Compare $P(X \geq 1)$ and $P(N \leq 9)$. Is there a reason these should be the same?

Problem 6. Let X be the number of rolls of a fair 10-sided fair die until a 5 or a 7 appears.

(a) Give the probability mass function of X.

(b) Compute $E[X]$, $\mathrm{Var}(X)$, and σ (the standard deviation of X).

[1] The tanuki (nyctereutes viverrinus) is a mammal native to Japan which resembles a reddish dog or a raccoon.