Math 154 Homework #5

Spring 2023

Due date: **11:59pm** Pacific Time on **Wed, May 17** (via Gradescope)

On the first page of your work, please write a list of everyone with whom you collaborated on this assignment, as well as any outside sources you consulted, apart from the textbook, your notes, and the course staff. If you did not collaborate with anyone, please explicitly write, "No collaborators."

Problem 1. Let G be a bipartite graph with parts A and B, where $|A| = |B| = n$. If $\delta(G) \geq n/2$, prove that G has a perfect matching.

Problem 2. A standard deck of 52 playing cards is shuffled and then dealt into 13 piles of 4 cards each. Prove that regardless of how the deck is shuffled, there is always a way to divide the cards into 4 groups of 13 cards each, where each group has one card from every pile, and one card from each of the 13 possible ranks $(A, 2, 3, \ldots, 10, J, K, Q)$.

Example: (the rows represent the original piles, and the colors are the groups we divide into)

3♣	4♥	J♠	9♣
J♥	2♣	4♦	5♣
6♦	6♥	6♣	Q♦
7♥	K♣	5♠	6♠
9♠	8♥	10♠	9♦
Q♣	7♦	3♠	9♥
5♦	A♦	J♣	5♥
A♥	7♠	2♠	K♦
K♥	4♠	3♦	4♣
3♥	A♠	10♣	Q♥
A♣	7♣	J♦	2♥
10♥	Q♠	10♦	K♠
8♦	2♦	8♣	8♠

Problem 3. For each of the graphs shown below, assign a minimum-size edge-coloring, and show that no edge-coloring with fewer colors exists.
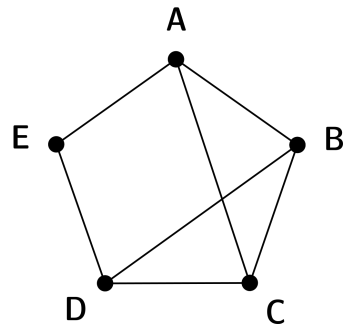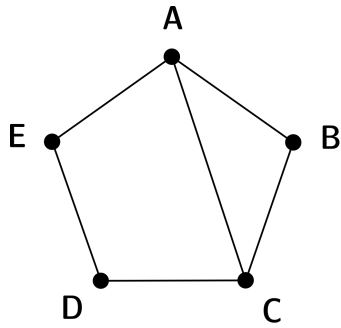
Problem 4. If G is a k-regular graph with an odd number of vertices, show that $\chi'(G) = k + 1$.