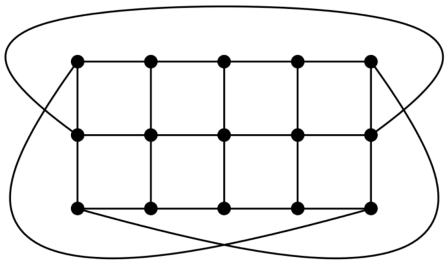
Math 154 Homework #7

Spring 2022

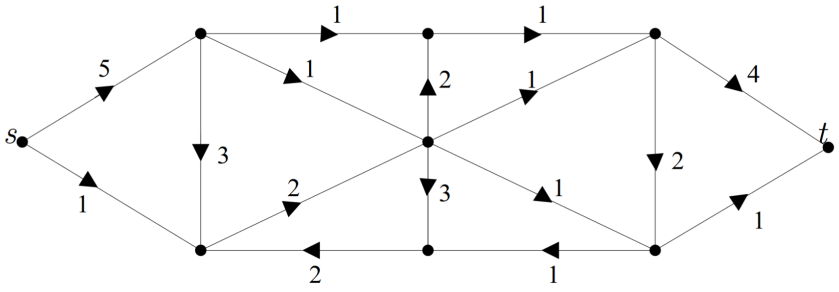
Due date: **11:59pm** Pacific Time on **Thu, May 26** (via Gradescope)

On the first page of your work, please write a list of everyone with whom you collaborated on this assignment, as well as any outside sources you consulted, apart from the textbook, your notes, and the course staff. If you did not collaborate with anyone, please explicitly write, "No collaborators."

Problem 1. Determine whether the graph G below is planar or not planar. If it is planar, prove it by explicitly drawing a planar embedding of G. If it is not planar, use Kuratowski's Theorem: identify a subgraph that is a subdivision of K_5 or $K_{3,3}$, and draw G with that subgraph clearly highlighted.



Problem 2. Use the Ford-Fulkerson Algorithm to find a maximum st-flow in the network shown below. Show every step of the algorithm: at each step, specify on which path you are augmenting the flow (drawing/highlighting arcs is okay), and by how much. To certify that you have found a maximum flow, specify a cut whose capacity is equal to the value of the flow you have found.



Problem 3. Let G be a directed graph with source s and sink t. Suppose F is a set of arcs after whose deletion there is no flow of positive value from s to t. Prove that F contains a cut.

Problem 4. In this class, we have only discussed directed graphs with a single source and a single sink; however, this is not an accurate model of many real-world directed networks (e.g., in a residential water-distribution network, every home is a sink).

(a) Describe how a multi-source, multi-sink network can be transformed into a network with a single source and a single sink, so that the methods we have learned in this class can be used to analyze it.

More specifically: given a directed network G with a set Σ of sources and a set T of sinks, and where the arcs have capacities, construct a network G' with a single source s and a single sink t in which the maximum value of a flow is the same as the maximum value of a flow in G. (The value of a flow in G is the net flow leaving Σ.)

(b) Use your transformation to solve the problem of finding the maximum total flow from the two sources to the three sinks in the network shown below. To certify that you have found a maximum flow in your transformed (single-source, single-sink) network, specify a cut whose capacity is equal to the value of the flow you have found.