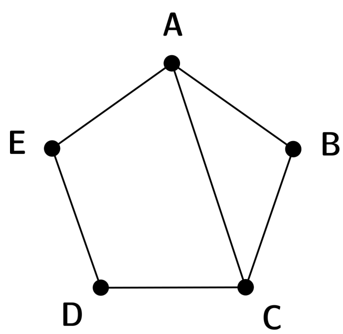
Math 154 Homework #6

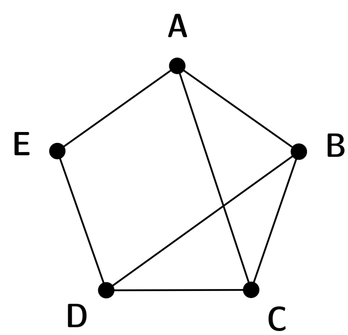
Spring 2022

Due date: **11:59pm** Pacific Time on **Thu, May 19** (via Gradescope)

On the first page of your work, please write a list of everyone with whom you collaborated on this assignment, as well as any outside sources you consulted, apart from the textbook, your notes, and the course staff. If you did not collaborate with anyone, please explicitly write, "No collaborators."

Problem 1. For each of the graphs shown below, assign a minimum-size edge-coloring, and show that no edge-coloring with fewer colors exists.





Problem 2. The **complement** $\overline{G}$ of a graph G is the graph on the same vertex set where $\{u, v\} \in E(\overline{G})$ if and only if $\{u, v\} \notin E(G)$. (In other words, to obtain the complement $\overline{G}$ of G, we fill in all the edges missing from G to form a complete graph, then delete the edges originally present in G.)

(a) Prove that, for every graph G on 11 or more vertices, at most one of G and $\overline{G}$ can be planar. *(If you get stuck, there is a hint on the last page.)*

(b) Give an example of a graph G on 11 or more vertices where both G and $\overline{G}$ are nonplanar. *(Note: you should show that they are not planar. If it is helpful, you may use without proof the fact that K_5 and $K_{3,3}$ are nonplanar.)*

Problem 3. Show that every triangle-free planar graph is 4-colorable.
(Note: do not use the 4-Color Theorem! And if you get stuck, there is a hint on the last page.)

Problem 4. (Bonus – NOT TO BE TURNED IN) The **Erdös-Renyi random graph** $G(n, \frac{1}{2})$ is a graph on n vertices constructed as follows: between each pair of vertices, we place an edge independently with probability $\frac{1}{2}$. (Note: this is one way of choosing a graph uniformly at random from among all graphs on n vertices.)

Show that $G(n, \frac{1}{2})$ is nonplanar with probability approaching 1 as $n \to \infty$.

(If you've taken Math 180A before, you have all the tools to do this!)

Some small hints

Hint for Problem 2: How many edges and vertices does $\overline{G}$ have?

Hint for Problem 3: In class, we showed that every simple planar graph has a vertex of degree ≤ 5, and then we used that to show simple planar graphs are 6-colorable. You may find a similar strategy useful!