Getting FTC II from FTC I

If f is a continuous function on $[a,b]$, we want to show that

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

for <u>any</u> antiderivative F of f.

We <u>do</u> know that f has at least one antiderivative, namely: $A(x) = \int_a^x f(t)\,dt$.

<u>Why?</u> Because $A'(x) = f(x)$, by FTC I

For <u>this</u> antiderivative, we can check that FTC II is true:

$$A(b) - A(a) = \int_a^b f(t)\,dt + \int_a^a f(t)\,dt \leftarrow \text{from definition of } A(x)$$

$$= \int_a^b f(t)\,dt + 0 \leftarrow \text{property of integrals (area under one point is 0)}$$

$$= \int_a^b f(t)\,dt$$

And this is what we wanted! This says that FTC II is true for the <u>specific</u>

antiderivative $A(x)$. But what about <u>other</u> antiderivatives $F(x)$ of $f(x)$?

If $F(x)$ is any other derivative of $f(x)$, we know it can be written as

$$F(x) = A(x) + C$$

for some constant C. So

$$\begin{aligned} F(b) - F(a) &= (A(b) + \cancel{C}) - (A(a) + \cancel{C}) \\ &= A(b) - A(a) \\ &= \int_a^b f(t)\,dt \end{aligned}$$

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And this is precisely FTC II!