SUMMARIES

[1] Zabrocki *On the action of the Hall-Littlewood vertex operator*

There is a classical operator $\mathbf{H_k}$ on symmetric polynomials which is characterized by the fact that the Hall-Littlewood polynomial $\tilde{H}_\mu(x;t)$ may be written in the form $\tilde{H}_\mu = \mathbf{H}_{\mu_1}\mathbf{H}_{\mu_1}\cdots\mathbf{H}_{\mu_1} 1$. In his thesis Zabrocki studies the action of $\mathbf{H}_k$ on the Schur function basis. His main result is a completely combinatorial description of this action. Using this result he is then able to give a new proof of the the Lascoux-Schützenberger "charge" interpretation of the Kostka-Foulkes polynomials. We should note that the original proof of this important result was technically very difficult and was accessible only to a handful of experts. Zabrocki's proof is relatively much simpler and makes the result accessible to a wider audience.

[2] Morse *Some classical expansions for Sahi-Knop and Macdonald polynomials,*

In recent simultaneous work Sahi and Knop introduced, for each composition α, a non-homogeneous non-symmetric polynomial $G_\alpha(x_1, x_2, \ldots, x_n; q, t)$ whose highest homogeneous component gives the non symmetric Macdonald [11] polynomial $E_\alpha(x_1, x_2, \ldots, x_n; q, t)$. Macdonald shows that for any composition α that rearranges to a partition λ, an appropriate Hecke algebra symmetrization of E_α yields the Macdonald polynomial $P_\lambda(x; q, t)$. In the original papers all these polynomials are only shown to exist. No explicit expressions are given relating them to the more familiar classical bases. The basic discovery here is that $G_\alpha(x_1, x_2, \ldots, x_n; q, t)$ appears to have surprisingly elegant expansions in terms of the polynomials $Z_\alpha(x_1, x_2, \ldots, x_n; q) = \prod_{i=1}^n (x_i; q)_{\alpha_i}$. In this paper contains the first results obtained in the problem of determining the connection coefficients relating the bases $\{G_\alpha\}_\alpha$ and $\{Z_\alpha\}_\alpha$. In particular we have here a complete solution to the problem for $n = 2$. The proofs rely heavily on the theory of basic hypergeometric series and reveal a deep connection between this classical subject and the theory of Macdonald polynomials.

[4] Morse *Recursions and Explicit formulas for particular n-variable Sahi-Knop and Macdonald polynomials*

The top homogeneous components of the Sahi-Knop polynomials $G_\alpha(x; q, t)$ are the non-symmetric Macdonald polynomials, $E_\alpha(x; q, t)$. An appropriate Hecke algebra symmetrization of E_α yields the Macdonald polynomials, $P_\lambda(x; q, t)$. The search for explicit formulas for the polynomials $G_\alpha(x; q, t)$ led to the main results of this paper. In particular, we have a complete solution for the case $G_{(k,a,\ldots,a)}(x; q, t)$. A remarkable by-product of the proofs is the discovery that these polynomials satisfy a recursion on the number of variables.

[5] Morse *Sahi-Knop and Macdonald polynomials Related to q-Ultraspherical functions and basic hypergeometric series*

It is shown here that the formula for the Sahi-Knop polynomials $G_{a,b}(x, y; q, t)$ derived in [2] and [3] when expressed in terms of the basis $(x; q)_a(x; q)_b$ yields an explicit expression for the Macdonald polynomials E_α and P_λ, in terms of the q-Ultraspherical polynomials of Askey and Ismail. The proof of this formula reveals that there is a deep connection between the theory of Macdonald polynomials and the classical theory of hypergeometric series and

[6] Morse (with A. Lascoux and L. Lapointe) *Classical operators and Macdonald polynomials*

The basic result here is a beautiful and remarkably simple expansion for the image of the Schur function basis, $S_\lambda\left[X\frac{(t-1)}{(q-1)}\right]$ under the action of any of the Macdonald operators. This result is then used to obtain completely explicit λ−ring expansions for the Macdonald polynomials in any number of variables in terms of this basis as well as in terms of the ordinary Schur basis, $S_\lambda[X]$.

[11] Garsia (with F. Bergeron) *Science Fiction and Macdonald Polynomials*

This work studies the remarkable relationships that hold among certain m-tuples of the Garsia-Haiman modules $\mathbf{M}_\mu$ and corresponding elements of the Macdonald basis. We recall that in [18], $\mathbf{M}_\mu$ is defined for a partition $\mu \vdash n$, as the linear span of derivatives of a certain bihomogeneous polynomial $\Delta_\mu(x,y)$ in the variables $x_1, x_2, \ldots, x_n$, $y_1, y_2, \ldots, y_n$. It has been conjectured in [18], $\mathbf{M}_\mu$ has $n!$ dimensions and that its bigraded Frobenius characteristic is given by the symmetric polynomial $\tilde{H}_\mu(x;q,t) = \sum_{\lambda \vdash n} S_\lambda(X)\tilde{K}_{\lambda\mu}(q,t)$ where the $\tilde{K}_{\lambda\mu}(q,t)$ are related to the Macdonald q,t-Kostka coefficients $K_{\lambda\mu}(q,t)$ by the identity $\tilde{K}_{\lambda\mu}(q,t) = K_{\lambda\mu}(q,1/t)t^{n(\mu)}$ with $n(\mu)$ the x-degree of $\Delta_\mu(x;y)$. Using this conjectured relation we can translate observed or proved properties of the modules $\mathbf{M}_\mu$ into identities for Macdonald polynomials. Computer data has suggested that as ν varies among the immediate predecessors of a partition μ, the spaces $\mathbf{M}_\nu$ behave like a boolean lattice. The same appears to holds true when ν varies among the immediate successors of μ. Combining this property with a number of observed facts and some "heuristics" we have been led to formulate a number of remarkable conjectures about the Macdonald polynomials. In particular we obtain a representation theoretical interpretation for some of the symmetries that can be found in the computed tables of q,t-Kostka coefficients. The expression "Science Fiction" here refers to a package of "heuristics" that we use to describe relations amongst the modules $\mathbf{M}_\nu$. These heuristics are purely speculative assertions that are used as a convenient guide to the construction of identities relating the corresponding bigraded Frobenius characteristics. Nevertheless computer experimentation reveals that these assertions are "generically" correct. Moreover, the evidence in support of the symmetric function identities we have derived from them are overwhelming. In particular, we show that various independent consequences of our heuristics lead to the same final identities.

[12] Garsia (with F. and N. Bergerons, M. Haiman and G. Tesler) *Lattice Diagram Polynomials and Extended Pieri Rules*

The lattice cell in the $i+1^{st}$ row and $j+1^{st}$ column of the positive quadrant of the plane is denoted (i,j). If μ is a partition of $n+1$, we denote by μ/ij the diagram obtained by removing the cell (i,j) from the (French) Ferrers diagram of μ. We set $\Delta_{\mu/ij} = \det \| x_i^{p_j} y_i^{q_j} \|_{i,j=1}^n$, where $(p_1,q_1), \ldots, (p_n,q_n)$ are the cells of μ/ij, and let $\mathbf{M}_{\mu/ij}$ be the linear span of the partial derivatives of $\Delta_{\mu/ij}$. The bihomogeneity of $\Delta_{\mu/ij}$ and its alternating nature under the diagonal action of S_n gives $\mathbf{M}_{\mu/ij}$ the structure of a bigraded S_n-module. We conjecture that $\mathbf{M}_{\mu/ij}$ is always a direct sum of k left regular representations of S_n, where k is the number of cells that are weakly north and east of (i,j) in μ. We also make a number of conjectures describing the precise nature of the bivariate Frobenius characteristic of $\mathbf{M}_{\mu/ij}$ in terms of the theory of Macdonald polynomials. On the validity of these conjectures, we derive a number of surprising identities. In particular, we obtain a representation theoretical interpretation of the coefficients appearing in some Macdonald Pieri Rules.

[13] D. Little

This paper deals with the power series expansion of the product $F_m(q) = \prod_{n>m}(1-q^n)$. This expansion is readily obtained from an identity of Sylvester and the latter, in turn, is given a relatively simple combinatorial proof. Nevertheless, the problem remains to give a combinatorial explanation for the massive cancellations which produce the final result. The case $m=0$ is clearly explained by Franklin's proof of the Euler Pentagonal Number Theorem. Efforts to extend the same mechanism of proof to the general case $m>0$ lead us here to the discovery of a remarkable extension of the Franklin involution which beautifully explains all the components of the final expansion.

[14] Garsia (with F. Bergeron) *Identities and conjectures for a remarkable operator on Symmetric Polynomials.*

The main ingredient in this paper is the symmetric function basis $\{\tilde{H}_\mu(x;q,t)\}_\mu$, where for each partition μ we set $\tilde{H}_\mu(x;q,t) = \sum_\lambda S_\lambda(x)\tilde{K}_{\lambda\mu}(q,t)$. Here $S_\lambda(x)$ denotes a Schur function and $\tilde{K}_{\lambda\mu}(q,t)$ are slightly modified Macdonald q,t-Kostka coefficients. We set $T_\mu = \prod_{s\in\mu} t^{l'(s)}q^{a'(s)}$ where for a cell s of the (french) Ferrers diagram of μ we let $l'(s)$ and $a'(s)$ denote the number of cells of the diagram that strictly South and strictly East of s. A number of years ago we introduced the linear operator ∇ on symmetric polynomials defined by setting $\nabla\ \tilde{H}_\mu = T_\mu\ \tilde{H}_\mu$ for all partitions μ. Computer experimentations revealed a number of truly remarkable properties of this operator. The most fascinating is that its action of a Schur basis element S_λ is always either totally Schur positive or totally Schur negative. This means that up to a sign ϵ_λ which can be easily identified, we have expansions of the form $\nabla\ S_\lambda(x) = \epsilon_\lambda \sum_\rho S_\rho(x)\ C_{\rho,\lambda}(q,t)$ with $C_{\rho,\lambda}(q,t)$ a polynomial in q,t with positive integer coefficients. We identify here some of these coefficients and prove some identities yielding images by ∇ of several classical symmetric functions. One of the remarkable developments is that using ∇ we can introduce a bivariate version of Lagrange inversion which unifies in one single formula a number of heretofore disconnected q-analogues of Lagrange inversion.

[15] Garsia (with M. Haiman and G. Tesler) *Explicit Plethystic Formulas for the Macdonald q,t-Kostka Coefficients*

For a partition $\mu = (\mu_1 > \mu_2 > \cdots > \mu_k > 0)$ set $B_\mu(q,t) = \sum_{i=1}^k t^{i-1}\,(1+q+\cdots+q^{\mu_i-1})$. In [9] Garsia-Tesler proved that if γ is a partition of k and $\lambda = (n-k,\gamma)$ is a partition of n, then there is a unique symmetric polynomial $k_\gamma(x;q,t)$ of degree $\leq k$ with the property that $\tilde{K}_{\lambda\mu}(q,t) = k_\gamma[B_\mu(q,t);q,t]$ holds true for all partitions μ. The main result established there is that these polynomials have Schur function expansions of the form $k_\gamma(x;q,t) = \sum_{|\rho|\leq|\gamma|} S_\lambda(x)\ k_{\rho,\gamma}(q,t)$ where the $k_{\rho,\gamma}(q,t)$ are polynomials in $q,t,1/q,1/t$ with integer coefficients. This result had a number of important consequences. To begin with it yielded the first proof of the Macdonald polynomiality conjecture. Secondly, it led to a proof (in [10]) of the positivity conjecture for the $\tilde{K}_{\lambda\mu}(q,t)$ for any λ of the form $\lambda = (m \geq 2, 1^m)$ and arbitrary μ. Last but not least it permitted the construction of extensive tables of the $\tilde{K}_{\lambda\mu}(q,t)$ and verify the Macdonald positivity conjecture for all partitions of $n \leq 14$. However in [9] the polynomials $k_\gamma(x;q,t)$ were only shown to exist by exhibiting an algorithm that permitted their construction. In this paper we show that the polynomials $k_\gamma(x;q,t)$ may be given a very simple explicit expression in terms of the operator ∇ studied in [12]. This important development allows a deeper study of these polynomials as well as the coefficients $\tilde{K}_{\lambda\mu}(q,t)$. Further consequences of these formulas are presently under investigation.

[16] Garsia (with F. Bergeron and G. Tesler) *Multiple left regular representations associated with alternants of the symmetric Groups,*

The experimental work that led to the results presented in [12] led to the conjecture that the linear span of derivatives of any alternant in $x_1, x_2, \ldots, x_n$ affords a multiple of the left regula representation of S_n. In this work we prove the conjecture and come to the remarkable discovery that the proof carries out almost verbatim for all groups generated by real or complex reflections. What is surprising is that in the case of alternants in $x_1, x_2, \ldots, x_n; y_1, y_2, \ldots, y_n$, under the diagonal action, we can only conjecture that the analogous result holds true only for alternants obtained by antisymmetrizing a single monomial. The paper contains a number of examples supporting this conjecture as well as examples of more general alternants in $x_1, x_2, \ldots, x_n; y_1, y_2, \ldots, y_n$ whose corresponding span of derivatives under the diagonal *does not* afford a multiple of the left regular representation of S_n. These developments reveal that there is a mysterious mechanism which governs diagonal actions which is yet to be undestood and should be the subject of further investigations.