MATH 20D Spring 2023 Lecture 4.

Implicit Solutions and Separable ODE's.

Outline

1. Implicit Solutions
2. Separation of Variables

Announcements

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- You must fill out the **Commencement of Academic Activity Survey** which is available via the *Quizzes* tab in Canvas. Please do this is as soon as possible, but no later than Friday this week.

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2 Separation of Variables

Definition

Let $I \subseteq \mathbb{R}$ be a domain and consider an n-th order ODE

$$F(t, y(t), \ldots, y^{(n)}(t)) = 0, \qquad (t \in I). \tag{1}$$

*An **explicit solution to (1) on** I is a function $\phi \colon I \to \mathbb{R}$ such that if $t \in I$ then*

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The equation $x^2 + y^2 = R^2$ defines two explicit solutions to the ODE

$$\frac{dy}{dx} = \frac{-x}{y}$$

on the interval $I = (-R, R)$.

Example

Let $C \in \mathbb{R}$ be constant. Show that the relation

$$\log|y| - \log|1-y| = x + C \tag{2}$$

defines an implicit solution to the ODE

$$\frac{dy}{dx} = y(1-y).$$

You may assume (2) defines y implicitly as a function of x.

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$$\frac{dP}{dt} = rP(1 - \frac{P}{K})$$

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- Recall that last time we discussed the **logistics equation**

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- The example above is the special case when $r = K = 1$.

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- $\frac{dy}{dx} = xy$ is **separable** but $\frac{dy}{dx} = 1 + xy$ is **not separable**.

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Using the method of separation of variables, solve the IVP

$$\frac{dy}{dx} = -xy$$

subject to the initial condition **(a)** $y(0) = 0$ and **(b)** $y(0) = 1$

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where $C = H(y_0) - G(x_0)$, $H'(y) = 1/p(y)$, *and* $G'(x) = g(x)$.

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Step 3: If possible, simplify the **implicit solution** to obtain an **explicit solution.**

Example

Using the method of separation of variables, solve the IVP

$$\frac{dy}{dt} = y(1-y)$$

subject to the initial condition **(a)** $y(0) = 1$, **(b)** $y(0) = 1/2$, and **(c)** $y(0) = 2$.