MATH 20C
WINTER 2020
SECTION D00 (MANNERS)

HOMEWORK – WEEK 9

Due by 2359 (11:59 PM) on Sunday March 8. Hand in via Gradescope.

For problem 0, credit is awarded for any honest response, not for the amount of work undertaken.

For problems 1, 2, 3 and 4, you *must* give a fully written-out solution showing all your working and justification. Stating the correct answer, by itself, will earn no credit.

0. Do the following textbook problems. *Do not turn them in*, but provide a list here of those for which you wrote down solutions.

§4.1: 3,7
§4.2: 3
§5.1: 9, 13
§5.3: 1, 3, 9

(*1 points*)

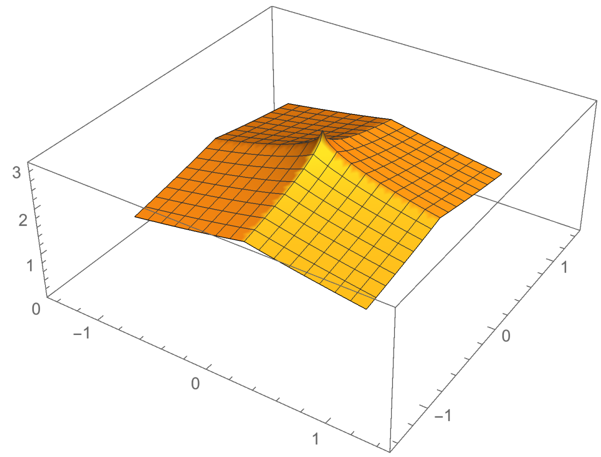
1. Consider the path $\vec{c}(t) = \big(2t, t^2, \log(t)\big)$ (defined for $t > 0$). Here $\log(t)$ is the natural logarithm. Find the arc length of $\vec{c}(t)$ between $t = 1$ and $t = 2$.

(*6 points*)

2. My house occupies the rectangle $[-1, 1] \times [-1, 1]$ in the x, y plane (measured in meters – it's a small house), and the height of the roof above the ground at position (x, y) is given (in meters) by the function

$$h(x, y) = 3 - \sqrt{|x| + |y|}.$$

Here is a picture of the roof of my house.



The inside of the house [1] therefore consists of all points

$$\{(x, y, z) \colon -1 \le x \le 1,\ -1 \le y \le 1,\ 0 \le z \le f(x, y)\}.$$

If (hypothetically) my house were entirely filled with water, from the ground to the roof, how many cubic meters of water would it hold?

(*6 points*)

3. Let R be the rectangle $R = [-1, 1] \times [-1, 2]$. Show that

$$1 \le \int\int_R \frac{1}{1 + x^2 + y^2}\, dx\, dy \le 6.$$

(*6 points*)

4. Let D be the triangular region with vertices $(0, 0)$, $(0, 2)$, $(2, 0)$. Evaluate the double integral

$$\int\int_D xy\, dA.$$

(*6 points*)

[1] Assuming there is no basement, etc.