MATH 20C
WINTER 2020
SECTION D00 (MANNERS)

Homework – week 6

Due by 2359 (11:59 PM) on Sunday February 16. Hand in via Gradescope.

For problem 0, credit is awarded for any honest response, not for the amount of work undertaken.

For problems 1, 2 and 3, you *must* give a fully written-out solution showing all your working and justification. Stating the correct answer, by itself, will earn no credit.

0. Do the following textbook problems. *Do not turn them in*, but provide a list here of those for which you wrote down solutions.

§2.5: 3, 7, 13, 33
§2.6: 1, 7, 12

(*1 points*)

1. There are two functions $f_1 \colon \mathbb{R}^3 \to \mathbb{R}$ and $f_2 \colon \mathbb{R}^3 \to \mathbb{R}$. I know very little about them, except that

$$f_1(1,2,3) = 4 \qquad f_2(1,2,3) = 7$$
$$\nabla f_1(1,2,3) = (-1,1,2) \qquad \nabla f_2(1,2,3) = (3,2,1).$$

I define $f \colon \mathbb{R}^3 \to \mathbb{R}^2$ by $f(x,y,z) = \big(f_1(x,y,z), f_2(x,y,z)\big)$. Also, $g \colon \mathbb{R}^2 \to \mathbb{R}^2$ is the function $g(a,b) = (a+b^2, ab)$.

(a) Let $h(x,y,z) = f_1(x,y,z)f_2(x,y,z)$ (the product). Compute $\nabla h(1,2,3)$.

(b) Write down the derivative matrix $Df(1,2,3)$, and compute the derivative matrix $Dg(a_0,b_0)$ at a general point $(a_0,b_0) \in \mathbb{R}^2$.

(c) Let $F(x,y,z) = g(f(x,y,z))$. Compute the derivative matrix $DF(1,2,3)$.

(*6 points*)

2. Let $f(x,y) = x^2 + 2y^2$. Consider the level set $C = \{(x,y) \in \mathbb{R}^2 \colon f(x,y) = 3\}$.

(a) Sketch C.

(b) Compute $\nabla f(1,1)$, and the equation of the tangent line to C at the point $(1,1)$.

(*6 points*)

3. Find the tangent plane to the surface $x^2 + 2y^2 + 3xz = 10$ at the point $(1, 2, 1/3)$.

(*6 points*)