MATH 142B
SPRING 2020
SECTION B00 (MANNERS)

Homework – week 5

Due by 2359 (11:59 PM) on Tuesday May 5. Hand in via Gradescope.

You may discuss these problems among yourselves, but your final submitted solutions must be written by you alone. You may not receive direct assistance on these problems from the internet.

1. Suppose $f\colon \mathbb{R} \to \mathbb{R}$ is a differentiable function, $A < B$ are real numbers, $f'(A) > 0$ and $f(B) < f(A)$. Prove that there exists $x \in (A, B)$ such that $f'(x) = 0$.

[**Hint:** consult the proof of Rolle's theorem and its variants.]

2. (a) Let $t > 0$ be a real number. Show that

$$\frac{t}{1+t} \leq \log(1+t) \leq t.$$

[**Hint:** apply the Mean Value Theorem to $f(x) = \log(1+x)$ on $[0, t]$.]

(b) Prove that for $a > 0$,

$$\lim_{n\to\infty} (1 + a/n)^n = e^a.$$

You may use without proof that exp is a continuous function.
[**Hint:** use part (a) and the squeeze lemma to find $\lim_{n\to\infty} n \log(1 + a/n)$.]

If you like, you can repeate this exercise for $t < 0$.

3. (a) Suppose $g\colon \mathbb{R} \to \mathbb{R}$ is an increasing function. Let $a \in \mathbb{R}$, and suppose that for all $\varepsilon > 0$ there exist $x < a$ with $g(x) > g(a) - \varepsilon$, and $y > a$ with $g(y) < g(a) + \varepsilon$. Prove that g is continuous at a.

(b) Suppose $f\colon \mathbb{R} \to \mathbb{R}$ is a differentiable function and that f' is a monotone increasing function on $\mathbb{R}$. Prove that f' is continuous.