MATH 142B
SPRING 2020
SECTION B00 (MANNERS)

Homework – week 3

Due by 2359 (11:59 PM) on Tuesday April 21. Hand in via Gradescope.

You may discuss these problems among yourselves, but your final submitted solutions must be written by you alone. You may not receive direct assistance on these problems from the internet.

1. For each of the following sequences of functions $f_n\colon X \to \mathbb{R}$, determine whether it converge uniformly on the domain X.

(a) $\sum_{n=0}^{\infty}(x/n)^{100}\frac{\sin(nx)}{1-x/2}$ on $X=(-1,1)$;

(b) $\sum_{n=0}^{\infty}(1/2-x)^n$ on $X=(-1/2,1/2)$;

(c) $\sum_{n=0}^{\infty}(-1)^n\sin(x)/n$ on $X=\mathbb{R}$.

2. For each of the following functions, determine (with justification) whether f is (i) continuous at $a=0$, and (ii) differentiable at $a=0$.

[These examples are intended to be argued directly from the definition. If you use other tools, justify their use carefully.]

(a) $f(x)=\begin{cases}\frac{x}{|x|^{1/10}} & : x\neq 0\\ 0 & : x=0\end{cases}.$

(b) $f(x)=x\,|x|^{1/10}-x$.

3. Suppose that $g\colon\mathbb{R}\to\mathbb{R}$ is any function (not necessarily continuous) such that $g(x)\in[-1,1]$ for all $x\in\mathbb{R}$. Let $f(x)=x^2g(x)$.

(a) Prove that f is differentiable at 0.

(b) Let $a\in\mathbb{R}$, $a\neq 0$, be any nonzero real number. Suppose f is differentiable at a. Prove that g is differentiable at a.