# MATH 142B
# SPRING 2020
# SECTION B00 (MANNERS)

## Homework – week 2

Due by 2359 (11:59 PM) on Tuesday April 14. Hand in via Gradescope.

You may discuss these problems among yourselves, but your final submitted solutions must be written by you alone.

**1.** For each of the following sequences of functions $f_n \colon X \to \mathbb{R}$, (i) find a function $f$ such that $f_n \to f$ pointwise, and (ii) decide (with proof) whether $f_n \to f$ uniformly on $X$.

**(a)** $f_n = \frac{1}{1+x^{2n}}$, $X = [-2, 2]$.

**(b)** $f_n = \frac{1}{1+x^{2n}}$, $X = [2, 3]$.

**(c)** $f_n = \frac{1}{1+x^{2n}}$, $X = (1, 3)$.

**2.** Given an example of a sequence of continuous functions $f_n \colon [0, 1] \to \mathbb{R}$ such that $f_n \to 0$ pointwise but $f_n$ does not converge to 0 uniformly.

**3.** Let $X \subseteq \mathbb{R}$ be a set, and $f_n \colon X \to \mathbb{R}$ a sequence of functions. Suppose $f_n$ converges uniformly to some limit. Is $f_n$ necessarily uniformly Cauchy? Give either a proof or a counterexample.

**4.** Consider the power series

$$\sum_{n=0}^{\infty} x^n/n!.$$

We have seen that this converges for every $x \in \mathbb{R}$. Does this converge uniformly on $X = \mathbb{R}$? Justify your answer.