Math 132A Winter 1981 Fillmore

Midterm Due February 2

(1) Assume that the rate of deposit or removal of sand on the bed of a stream is proportional to the derivative of the velocity of the flow of water with respect to distance in the direction of flow.

a) Show that the height $u(x,t)$ of the sand in a stream bed, above an arbitrary zero level, satisfies the PDE

$$(h - u)^2 \frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} = 0 \quad ,$$

where $h(x,t)$ is the height of the water surface, x is distance in the direction of flow, and a is a constant.

b) Assume h is constant and find the solution (implicit) of this PDE which satisfies the initial condition $u(x,0) = f(x)$. Use the method of characteristics.

(2) For the equation $u_y = u_x^3$,

a) find the solution satisfying $u(x,0) = 2\, x^{3/2}$.

b) show that a solution differentiable for all x,y is linear.

(3) From Gustafson PDEs, work Problem 2 of Section 1.3 (p.14) on the Poisson representation of the solution of the heat equation.