

The Apollonius contact problem

Setting

$V = \mathbb{R}^2$ (col) Euclidean plane

Ω^3 signature $(-1++-1+)$ or $(2,3)$ in \mathbb{P}^4

Problem Find all line cycles in oriented contact with three given line cycles.

Solution Line cycles in oriented contact with $\langle \alpha \rangle$ in Ω^3 are: $\Omega^3 \cap \langle \alpha \rangle^\perp$.

Those in oriented contact with

$\langle \alpha \rangle$, $\langle \beta \rangle$, and $\langle \gamma \rangle$ are

$$(\Omega \cap \langle \alpha \rangle^\perp) \cap (\Omega \cap \langle \beta \rangle^\perp) \cap (\Omega \cap \langle \gamma \rangle^\perp)$$

$$= \Omega^3 \cap \underbrace{\langle \alpha, \beta, \gamma \rangle^\perp}_{\dim=1} = \Omega^3 \cap (\text{line}),$$

which consists of three solutions in general.

