MATH 151 - TOPICS IN GEOMETRY

Spring 1990: Lie Geometry

TOPICS: Selected topics from Sophus Lie's geometry of families of circles. Discussion of sphere geometry and contact transformations.

PREREQUISITE: A basic knowledge of Linear algebra (for example, Math 2E), a love of geometry, and a willingness to participate.

RECOMMENDED: A familiarity with modern algebra (for example, Math 100A or 103A).

FORMAT: Lectures and participation in problem solving.

TEXT: Insturctor's class notes.

MEETS: Tuesday and Thursday 10:00 - 11:50, USB 4020A.

INSTRUCTOR: J. Fillmore, AP&M' 5747, 534-2651.

NOTE: Math 151, with different topics, may be repeated for credit.

The subject is the part of Lie's "higher sphere geometry" dealing with circles in the Euclidean plane. Circles themselves and their pencils are the fundamental objects of study, not points. It subsumes (contains) the familiar inversive geometry of Möbius. The goal of this course is to explain the basic ideas of this geometry in the familiar terms of Euclidean geometry and linear algebra.

The subject, created by the Norwegian mathematician Sophus Lie in the 1870s, has been dormant since the 1940s, but is recently enjoying a resurgence of interest in connection with differential geometry. Some theorems discovered as recently as twenty years ago will be discussed and proved completely.

As there exists no current text on the subject, sources will be journal references and course notes from the instructor.

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Books on reserve in Science and Engineering Library

Seidenberg, A. (Abraham), 1916-
Lectures in projective geometry. Princeton, N.J., Van Nostrand [1962].
Series title: The University series in undergraduate mathematics.
UCSD S & E QA471 .S458
Overview of projective geometry with emphasis on linear algebra.
Especially useful: Chs. V to XIII on analytic projective geometry.

Pedoe, Daniel.
A course of geometry for colleges and universities [by] D. Pedoe.
London, Cambridge University Press, 1970.
UCSD S & E QA445 .P43
Emphasis on representational geometry and inversive geometry.
Easy reading. Near to the approach used in the course.
Ch. IV. The representation of circle by points in space of three dimensions.
Ch. VI. Mappings of the inversive plane.
Ch. VIII. The projective geometry of n dimensions.
Ch. IX. The projective generation of conics and quadrics.

Behnke, Heinrich, 1898-
Fundamentals of mathematics / edited by H. Behnke ... [et al.];
translated by S. H. Gould. Cambridge, Mass. : MIT Press, [1974].
UCSD Central QA37.2 .B413 and UCSD S & E QA37.2 .B413 V.2
Many interesting articles on various geometries. Easy general reading.
Geometries of circles and Lie geometry discussed in:
Ch. 12. Erlanger program and higher geometry. Kunle and Fladt.

The Geometric Vein : the Coxeter Festschrift
/ edited by Chandler Davis, Branko Grunbaum, F.A. Sherk
with contributions by Patrice Assouad ... [et al.].
New York : Springer-Verlag, c1981.
UCSD S & E QA446 .G46 1981
Research articles and expositions. Geometries of circles and Lie geometry
closest to the approach used in the course:
I. M . Yaglom. On the circular transformations of Möbius, Laguerre, and Lie. pp. 345ff.
J. F. Rigby. The geometry of cycles and generalized Laguerre inversion. pp. 355ff.
Related article, more difficult reading:
J. B. Wilker. Inversive geometry. pp. 379ff.

Additional references - not on reserve. Please return for others when not being used.

Gruenberg, Karl W.
Linear geometry / K. W. Gruenberg, A. J. Weir. 2d ed.
New York : Springer-Verlag, 1977.
Series title: Graduate texts in mathematics ; 49.
UCSD S & E QA564 .G72 1977
Projective geometry using coordinate-free linear algebra.
Ch. II. Affine and projective geometry.
Ch. III. Isomorhisms.
Ch. V. Bilinear forms.

Coolidge, Julian Lowell, 1873-1954.
A treatise on the circle and the sphere. Bronx, N.Y., Chelsea Pub. Co. [1971].
UCSD S & E QA484 .C65 1971
Classic 1916 treatise on circles and spheres. Very difficult reading.
It is best to translate ideas to modern setting and then provide one's own proofs.

Math 151 Suggestions for term-paper topics
Spring 1990 related to Lie geometry

You will need to do some writing for this course. The minimal acceptable amount will be to write careful solutions to the routine exercises which are suggested from time to time.

A more interesting undertaking is a term paper on a topic suggested below or on a similar topic. You should discuss your choice of topic with Fillmore or Springer before embarking upon it, so that we can ascertain that it neither too trivial nor too difficult. Your writing is expected to be of university quality. If possible, you should type or word-process the manuscript. Writing mathematics is not easy, but you will find it rewarding.

Careful solutions to routine exercises

Cross ratios
- relations betweeen the many definitions of cross ratio
- in projective space without a quadric
- in projective space with a quadric
- relation to cross ratio used in the complex plane

Duality in geometry
- dual projective space and duality in linear algebra, row and column vectors
- linear projective maps, perspectivities, projectivites, collineations, correlations, polarities, others.
- connections with linear algebra
- consult several books on projective geometry

Line geometry and Plücker coordinates
- Plücker quadric in five dimensional projective with signature (+++---)
- relate to space of lines in three dimensional Euclidean space
- the "line-sphere transformation" for Lie geometry

Minkowski geometry (signature - +)
- general facts and theorems
- relation to special relativity
- Lie geometry based on Minkowski (quadric with signature (+-|-+-)
- Ref. Yaglom, A non-Euclidean geometry and ... This is on Galilean geometry with remarks on Minkowski geometry

(over)

Inversive distance
- connection with hyperbolic non-Euclidean geometry
- connection with deSitter space in general relativity
- connection with Möbius geometry
- connection with Lie geometry
- Ref. Coxeter and Greitzer, Geometry revisited, and papers by Coxeter

Stereographic projection
- relation to the Lie quadric
- classical "isotropic projection"
- Ref. F. Klein, Vorlesungen über höhere Geometrie

Classical Laguerre geometry in terms of Lie geometry
- papers by D. Pedoe
- classical artical by van der Waerden & Shmit (sp?)

Laguerre and Möbius geometry in terms of Lie geometry
- study of the "pseudo duality"
- Rigby artical in the Coxeter Festshrift

Extract a topic from the Gindikin article

The Arbelos problem
- Farey fractions
- the rational Euclidean plane
- use of Möbius transformations
- use of Lie transformations
- Refs.: Books on number theory

Computer program
- plane figures related to Lie geometry
- bunches, Lie configuration, Tyrrell-Powell
- three dimensional figures for the Lie quadric
- Refs.: Books on computer graphics