Principal directions

$\bar{X} = S_1 X^1 + S_2 X^2$ — principal direction at P.

$$\left(\begin{pmatrix} L & M \\ M & N \end{pmatrix} - \lambda \begin{pmatrix} E & F \\ F & G \end{pmatrix} \right) \begin{pmatrix} X^1 \\ X^2 \end{pmatrix} = 0 \qquad \lambda = \kappa_1 \text{ or } \kappa_2.$$

Then

$$- \underbrace{\begin{pmatrix} E & F \\ F & G \end{pmatrix}^{-1} \begin{pmatrix} L & M \\ M & N \end{pmatrix}}_{\begin{pmatrix} a^1_1 & a^1_2 \\ a^2_1 & a^2_2 \end{pmatrix}} \begin{pmatrix} X^1 \\ X^2 \end{pmatrix} = - \lambda \begin{pmatrix} X^1 \\ X^2 \end{pmatrix}$$

or

$$S_*(P)\, \bar{X} = -\lambda \bar{X}.$$

i.e. principal directions are eigenvectors of $S_*(P)$. or $S_*(P)\bar{X}$ & $\bar{X}$ are parallel vectors.

Rodrigues' formula

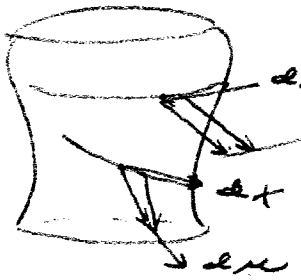
$X(t) = S(\alpha(t))$ line of curvature:

$$\underbrace{S_*(\dot{\alpha}(t))\, \frac{dX}{dt}(t)}_{\frac{d}{dt} N(\dot{\alpha}(t))} = -\lambda(t)\, \frac{dX}{dt}(t)$$

$$\Rightarrow \quad \frac{dN}{dt} + \lambda \frac{dX}{dt} = 0$$

$$\text{or} \quad dN + \lambda\, dX = 0.$$

Eg



- line of curvature

- not line of curvature.