Lecture 14. Lobachefsky geometry

Non Euclidean geometry

Classically: A geometry which violated Euclid's fifth postulate: Given a point P not on a line l there is exactly one line l' through P not meeting (parallel to) l.

Elliptic geometry

(R iemann). Every line l' through P meets l in one point. This is the metric geometry of the projective plane. Corresponds to Gauss curvature $K = \text{constant} > 0$.

Hyperbolic geometry

(Lobachefsky + Bolyai, and Gauss). There are infinitely many lines l' through P which do not meet l. This corresponds to Gauss curvature $K = \text{constant} < 0$. This is described in several models below.

Rmk Both elliptic and hyperbolic geometry can be defined axiomatically like Euclidean geometry.