of f we know that w splits the preimage of its left descendents from the preimage of its right descendents, contradicting the adjacency of u_1 and u_2. Hence $f(u_1)$ is a descendent of $f(u_2)$ or vice versa. It follows that for each w in C, S_w is separated from the rest of the graph in $T \setminus A_w$, since otherwise there would exist two vertices in T, neither of whose image would be a descendent of the other, and this violates the above result.

This fact can be used to prove an interesting analog of the necklace lemma for trees.

Theorem. *Let T be a tree on n vertices that are colored with k colors. Then there exists a constant $c > 0$ such that by removing at most $ck \log n$ vertices, one can partition the remaining connected components into two sets, each of which contains the same number of vertices in each color.*