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A Nice Genius

As a boy, Ron Graham, was small for his age and was never chosen to play on school teams. He never attended the same school for more than one year and did not graduate from high school. He did not take any mathematics courses during his first three years of college at The University of Chicago and eventually got a bachelor's degree in physics from the University of Alaska.

Today at age 61 Dr. Graham stands six feet-two inches and looks very much the athlete. He is regarded as one of the top mathematicians in the world. Graham is famous for his work in combinatorics, which won him election to the National Academy of Sciences. He is past-president of the American Mathematical Society, and he has just been named Chief Scientist of AT&T Laboratories, successor to the much venerated Bell Labs. In spite of his heavy responsibilities at AT&T, he manages to find time to pursue other interests, especially juggling and gymnastics. In college he was a California state trampoline champion. He also is highly skilled in Ping-Pong, tennis, bowling, and boomerang. Bungee trampolining is his latest interest.

Harvard mathematician Persi Diaconis who has collaborated with Graham many times, describes him as "a remarkably accomplished mathematician. Ron is always willing to help a struggling student or a colleague. He never leaves you hanging. He's a genius, but a nice genius."

Ron Graham, Chief Scientist of AT&T Labs, in unicycling gear—suit, tie, and unicycle. Photograph by Ché Graham.

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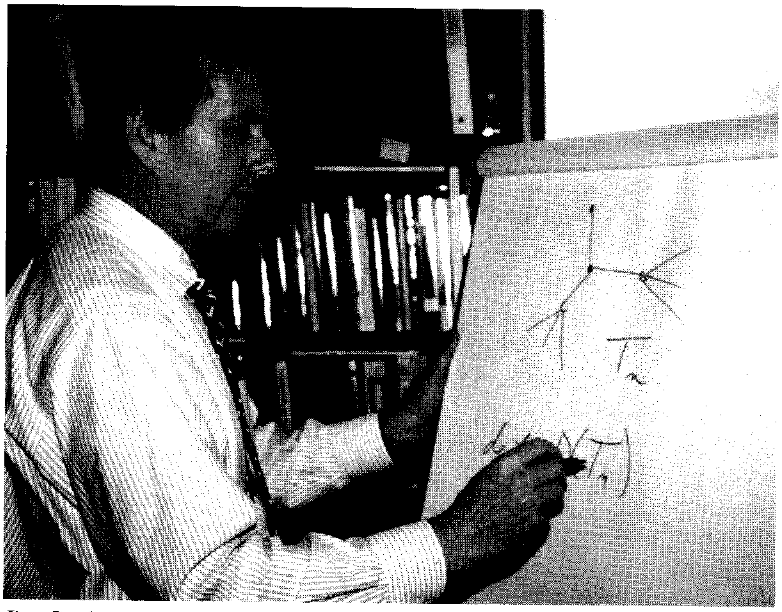
Diaconis remembers giving a talk about joint research that he and Graham had done. He ended his talk by saying "This problem is still unsolved." At that point, Graham, who was in the audience, stood up and gave a solution on the spot. The audience, thoroughly impressed, burst into applause, an unusual outpouring of emotion for a group of mathematicians.



Dr. Graham at work on a graph theory problem. Photograph by Carol Baxter.

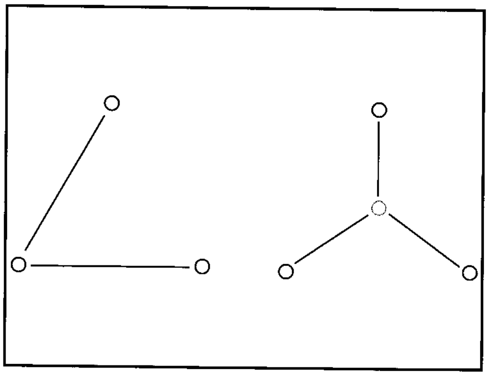
Calculus At Age 11

Ron Graham remembers liking mathematics a lot as a little boy. In fifth grade, Miss Smith showed him an algorithm for calculating square roots. He soon developed his own algorithm for calculating cube roots. His natural ability for the subject resulted in a high level of confidence. "By the time I got to seventh grade," he recalls, "I knew algebra and trigonometry and thought I could solve any problem given to me. But one day, Mr. Schwab gave me one I couldn't do. The problem was to find the size of a population of mice if it was known that the death rate was proportional to the size of the population. It took a knowledge of differential equations to solve it. He then gave me a book and told me that I would be able to solve the problem by the time I finished it. That book was *Calculus* by Granville, Smith, and Longley. He was right. By the end of the semester, I finished the book and had a solution. He also chose me to be on the school chess team. I owe a lot to Mr. Schwab."

Ron was only eleven years old at the time. He had skipped a few grades as his family moved around the country, primarily in California and back and forth to Georgia. Until he entered The University of Chicago when he was 15, he never attended any school for more than one year. Family stability was diminished in 1941 when Ron's father, an oilfield worker, enlisted in the Merchant Marine, and essentially left his wife and three children to fend for themselves. Young Ron had learned that superior academic achievement went a long way with teachers and peers. In his words, "I was a good kid and I learned to adapt."

Ron saw his father again one day six years later when he was delivering the *Berkeley Gazette.* He delivered papers both in the morning and in the evening. He remembers living in the housing projects and being poor.

Graham's tenacious pursuit of solutions has continued to this day. More often than not, he triumphs over problems, but after expending lots of time on one without significant progress, he will occasionally post a $100 reward for a solution. "Some problems are capable of driving me crazy," says Graham, "and I will pay just to be put out of my misery."



Adding a point (right) yields a shorter network.

Much of Graham's work has been in graph theory in support of AT&T's communications network. Telephone networks are examples of mathematical graphs. Telephone networks are replete with graph theory problems. The *shortest network problem* is one of the easiest of those problems to state but it is far from being easy to solve. The problem asks for the shortest network of line segments interconnecting an arbitrary set of, say, 100 points. The solution to this problem has frustrated the best mathematical minds and overwhelmed computers.

Adding Points

Sometimes adding points yields a shorter network. Take for example three points arranged at the vertices of an equilateral triangle. Any two sides of the triangle then give a solution. If, however, a point is added to the center of the triangle, then a shorter network results from joining that point to the vertices of the triangle. See figure.

Complete Disorder Is Impossible

It is his contribution to Ramsey theory, however, that mathematicians regard as his finest work. "Ramsey theory," according to Graham, "says that complete disorder is impossible. There is always structure somewhere." Ramsey theory shows that in any group of six or more people, three will either know one another or will all be strangers. It is somewhat harder to prove that in a group of 18 people there must be four who all know each other or are all strangers. The minimum size of the group needed to ensure that there are always five mutual friends or five mutual strangers is unknown.

In his junior year of high school, Graham's mother encouraged him to apply for a new scholarship program set up by the Ford Foundation that enabled talented students to enter certain universities before they finished high school in order to provide them more of a collegiate education before they might be drafted into military service. He scored high on the scholarship exams and was admitted to The University of Chicago. Since he had done especially well on the mathematics portion of the exam, he was exempted from further mathematics courses at Chicago. As a result, during his three years there, he took no mathematics, but he did discover the world of gymnastics and juggling. When he arrived at Chicago, he was still small for his age, but the right size for gymnastics.

Acrotheater

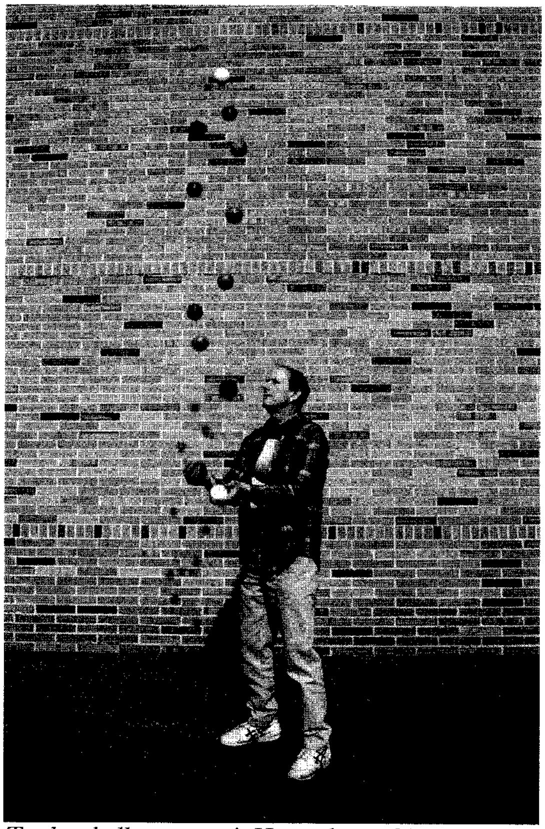
One of the classes that Graham enrolled in at Chicago was Acrotheater. It was a combination of dance, circus arts, and gymnastics. The teacher was E.F. (Bud) Beyer, a national gymnastics champion in 1935. Ron remembers the class with great fondness: "The class met twice a week. We would do shows in the Chicago area high schools and elsewhere; it was a recruiting device of sorts. At the end of the year we'd do a big show with music, make up—the whole nine yards. Acrotheater hooked me on gymnastics."

During his three years at Chicago, the undersized kid developed into a six foot two inch eighteen-year old. Juggling and the trampoline also developed into life-long interests for him during that period. The year after he left Chicago for the University of California, Berkeley, he later won the California Intercollegiate Trampoline championship. Some years later he was elected president of the International Jugglers Association.

After three years at Chicago, Graham transferred to Berkeley in 1951 as an electrical engineering major. "The problem was," he recalls, "that by then I was behind in math requirements and would not complete a degree in four years. At that time you could be drafted if you had not completed a degree in four years. So I enlisted in the Air Force and was stationed in Alaska." He arranged his schedule so that he attended classes by day at the University of Alaska and did his Air Force job in communications at night. He took more mathematics classes at Alaska, but ended up with a degree in physics in 1958 because Alaska was not yet accredited in mathematics.

The Berkeley System

In 1959, nine years after starting college, Graham returned to Berkeley to begin his doctoral studies—in mathematics! He remembers that his first semester at Berkeley was very challenging. "I didn't catch on to the 'system' for a while. In my first year, I walked into Chern's class on differential manifolds and on the first day things were moving pretty fast. And the next day, people in the front rows were saying, 'Come on, let's get on to the good stuff!' Chern thought he was moving too slowly for the class, so he really started ripping. It turned out that graduate students typically audited a course once or twice before taking it for credit. That was the system and it stimulated me to work hard."



Twelve balls at once! How about thirteen, Ron? Photograph by Ché Graham.

By 1962, Graham had thoroughly mastered the system and was the proud possessor of a PhD. His thesis on Egyptian fractions was supervised by the legendary number theorist D.H. Lehmer. An Egyptian fraction is a fraction having numerator 1, a so-called unit fraction. Thus 1/2, 1/7, 1/15 are Egyptian fractions. Notice that $3/5 = 1/2 + 1/10$. Now 2/7 is a bit harder to write as a sum of Egyptian fractions: $2/7 = 1/5 + 1/13 + 1/115 + 1/10465$. It's known that every fraction can be written as a finite sum of unit fractions. What if you restrict yourself to using fractions with odd denominators? The answer turns out to be yes, provided that fraction has an odd denominator. How about using distinct unit fractions, using only those that are only perfect squares. Graham showed that, again, the answer is yes, where it's reasonable. [1]

Graham's gymnastics interests continued unabated during his graduate student years. He and two other students formed a professional trampoline group, the Bouncing Bears, and earned money by performing at schools, supermarket openings, and the circus! Bungee trampolining is Graham's latest sport. In Bungee trampolining, a pair of Bungee cords are mounted over a trampoline and connected to a harness that is usually attached to a twisting belt, so that you can twist and somersault freely while being suspended by the two cords. "You go up about 30 feet above the surface, and if you miss, well, you get only bruised ribs. Eventually, you take off the harness and do the trick, unassisted. It's important, I think, to keep doing new things when you get into a rut, and start operating solely on reflexes."

Erdös

In 1963, Graham met the famous Hungarian mathematician Paul Erdös at a mathematics meeting in Colorado. Erdös, was one of the most prolific mathematicians in history with more than 1500 published papers to his credit. For more than fifty years, Erdös traveled from university to university with one small suitcase, gathering and sharing mathematical ideas with hundreds of mathematicians. Number theory and graph theory, Graham's specialities, were also the specialities of Erdös. In short order, they collaborated on a number of papers. As Erdös aged, Graham helped him tend to some of the basics of life—paying taxes, buying clothes, etc. Graham even set up an Erdös room in his home. Up until the time of his death in September of this year, Erdös frequently stayed with Graham and his wife Fan Chung.

Graham has vivid memories of his first meeting with Erdös. "I saw this rather senior guy of 50, already quite famous, playing Ping-Pong during one of the breaks. He asked me if I wanted to play, and I agreed. He absolutely killed me! I had played casual Ping-Pong, but I couldn't believe that this old guy had beaten me. So I went back to New Jersey and I got a machine that shoots out Ping-Pong balls at you. I bought a table, joined a club, started playing at Bell Labs, and in the state league. I eventually became the Bell Labs champion at Ping-Pong, and won one of the New Jersey titles. Finally, I reached the point where I thought I could play Erdös. Erdös, of course, was getting older, and I was playing more. When I could play him, I'd do it sitting down. I would sit in a tall chair, and let him start at 20. Sitting down in a chair is really not that big a disadvantage. You'd think that's going to handicap you. You can't get up from the chair. If you get up, you lose. It turns out that isn't that big of a disadvantage against somebody who's not such a serious competitor."

"As you work at Ping-Pong, you begin to understand the subtleties of the spin and the sound of Ping-Pong, so to speak. I always liked that aspect of it. The trouble is, it takes a certain investment to do that, and you can't do that for everything. And there's a certain carryover, but sometimes it's negative. What you do at Ping-Pong isn't necessarily good for what you do at tennis. It's like learning Chinese. If you learn Mandarin, that may or may not be helpful in learning Cantonese."

True Mark of Teaching

Graham's passion for learning is quite remarkable. He loved school because "I could learn something new, and I enjoyed especially teaching to someone else what I had learned and taking them one level beyond. It's very satisfying teaching someone a particular skill like juggling, the trampoline, or mathematics. But it's even better to take them to the next level where you teach them how to teach."

He flies through the air with the greatest of ease, the man on the bungee trampoline. Photograph by Ché Graham.

Graham claims that in the juggling world, there is a tradition that you teach people what you know, and they will in turn teach someone what they have been taught and what they've learned in addition. "I taught Tom Brown who in turn taught Joe Buhler how to juggle and he now is a better juggler than I am. That's a true mark of your teaching ability if you produce better students than you are."

After completing his doctorate at Berkeley in 1962, Graham joined the research staff of Bell Labs. He had been warned by academic mathematicians that he would be dead mathematically in three years if he went into industry. His election as President of the American Mathematical Society and as a member of the National Academy of Sciences argue against the prediction. He found the "open door" policy of Bell Labs especially stimulating and very conducive to cross fertilization with physicists, chemists, engineers, economists, and other mathematicians.

"That results in a different mind set to some extent," according to Graham, "and that's important for students to know if they hope to take jobs in industry. That's not easy to teach because most people in university settings aren't accustomed to working that way." He notes that doors at many universities aren't open so much.

One of Graham's goals as Chief Scientist of AT&T Labs is to mold programs which establish better exchanges with people in universities and the Labs. During his 34 years with AT&T, Graham has taught courses seven times in universities such as UCLA, CalTech, Stanford, Princeton, and Rutgers. He finds contact with students very helpful. "It's good for someone like me to stand up in front of good students who ask questions that you take for granted. They may say: 'Well, why do you do it this way?' "The answer might be: 'We always do it this way.' That forces you to think about what you've been doing and to question authority. I keep a saying on my refrigerator to remind me of staying open to new ideas. It says:

The Main Obstacle to Progress is Not Ignorance, But the Illusion of Knowledge.

When you think you know, that can blind you to the fact that you don't know. Once you know you don't know, then you might be more receptive to looking at other approaches."

Research And Arrows

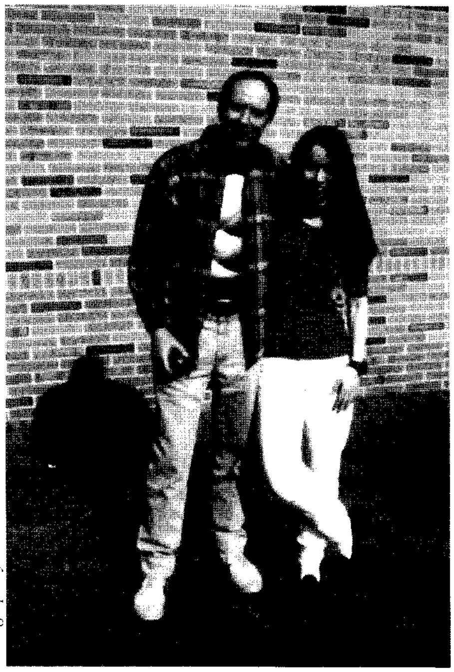
Graham took over as Chief Scientist of AT&T Labs in October at a transition point in the history of the company, which recently split into AT&T and Lucent Technologies. The quickening pace of technology has placed pressure on researchers to direct more of their efforts toward the business side. Graham's job, in part, is to foster the scientific excellence of the cultural careers of the researchers. He says that "I will make sure that they feel comfortable and are encouraged to pursue very fundamental ideas such as quantum computing that may not have any business impact for 20 years or more." He also notes that "pioneers are the people who have arrows in their chests, so you may not want to be out there too far ahead. At the same time, it's crucial to have the places and people who believe in fundamental research from the long-term viewpoint."

Graham at home with his spouse, mathematician Fan Chung of the University of Pennsylvania. Photograph by Ché Graham.

Graham is very much aware that in most businesses you often don't have years: the next model has to be out the door in a few months or the competition will blow you away. At the same time, someone somewhere has to be doing the basic research. Graham believes that most of that research will be done in universities, with some cooperation with industry. He also thinks that some large organizations such as AT&T are sufficiently forward looking, have the resources, and understand the necessity of doing basic research.

As our interview came to a close in Graham's office at the National Academy of Sciences, one of three offices that he maintains, I remarked that he probably would be glad to get home to New Jersey and relax after flying back from Washington. He said that he planned to relax by working on a new juggling trick of the site-swap type, and perhaps listen to some classical music. He explained that a site-swap is a way of taking a numerical pattern such as 3-4-5 and converting it to a juggling trick. The 3, 4, and 5 tell you how high the balls go, and the sequence tells you which hands you're to do it with. So the right hand throws the 3 the left hand 4, the 3 crosses, and the 4 doesn't. But the next hand, which is the right, throws a 5. Then this is repeated, and you've got to do all of this in real time.

He says that it's not simply a control issue, but that you have to know where to look. If you're juggling, say, five balls, you don't watch every ball all the way because there are too many balls and not enough eyes. You have to look at a given ball only briefly. Typically, you look at each ball at the top of the arc and get some information there. You mentally compute where it's going and put your hand there. Every throw is a little bit off, some off more than others, and juggling in large part is maintaining that radius of allowable mistakes, so to speak, in the "region" of stability."



Photograph by Ché Graham.

Cheryl Graham, Ron's daughter, is an accomplished photographer who likes juggling too.

When juggling five balls, the typical pattern for each of the balls is essentially the same. You see a very symmetrical and beautiful pattern. They're all crossing. In a typical pattern they cross at the top, so you watch right there. The other patterns, such as a site swap, where different hands are throwing different balls to different heights can be hard to follow: There's no single place to look. Different hands are throwing balls to different heights, and there's no good place to look. You kind of back off and look globally. It's better to focus on each ball, even though briefly. Graham says, "Well, that may not be the best way to juggle, but it kind of helps your normal juggling, because it teaches you something that you did when you were first learning, and you watch, you want to keep your eye on the ball, so to speak—on the balls—and you get lazy." For Dr. Graham, it's always been the case that if he goes more deeply into a subject, he appreciates it more. He says "To understand how juggling evolves, if it's not so hard for four balls, try five, or try it underwater, or while trampolining, or with your arms crossed."

Music, Understanding, And Magic

Graham also likes music very much. Naturally, the same question comes up—would he appreciate it more by going more deeply into it? If he really tried to understand the structures of what Beethoven, Bach, or Mozart had been doing, would he appreciate it more. He worries that it might somehow lose the magic: "It's as though you like to be impressed by a magic trick, but once you know how it's performed, the magic is gone."

"For example, I can flip a coin and be very 'lucky.' I can be extremely 'lucky.' I can, in fact, be perfectly 'lucky' if necessary. But once you understand how it happens, the mystery and the magic is gone. Would that happen with music? I don't know, but I have deliberately stayed away from seeking a deep understanding of music." Musicians tell me, "No, you'll appreciate it even more when you really understand how great these composers were."

One of the complaints about mathematics is that you couldn't really understand the beauty and elegance, the power unless you're another mathematician. How do you convey that to someone else? There's a certain truth in that although you can certainly show people to some extent the power or the elegance, and the surprise of some parts of mathematics.

In a few weeks, Graham will travel to the Disney Institute to participate in special mathematical programs for the general public. Dr. Graham, the mathematician, teacher, and athlete, continues to work at displaying a friendly side of mathematics to the public. He is, indeed, a nice genius. ■

Note

[1] The fraction needs to be in the union of the intervals $[0, \pi^2/6-1)$ and $[1, \pi^2/6-1)$.