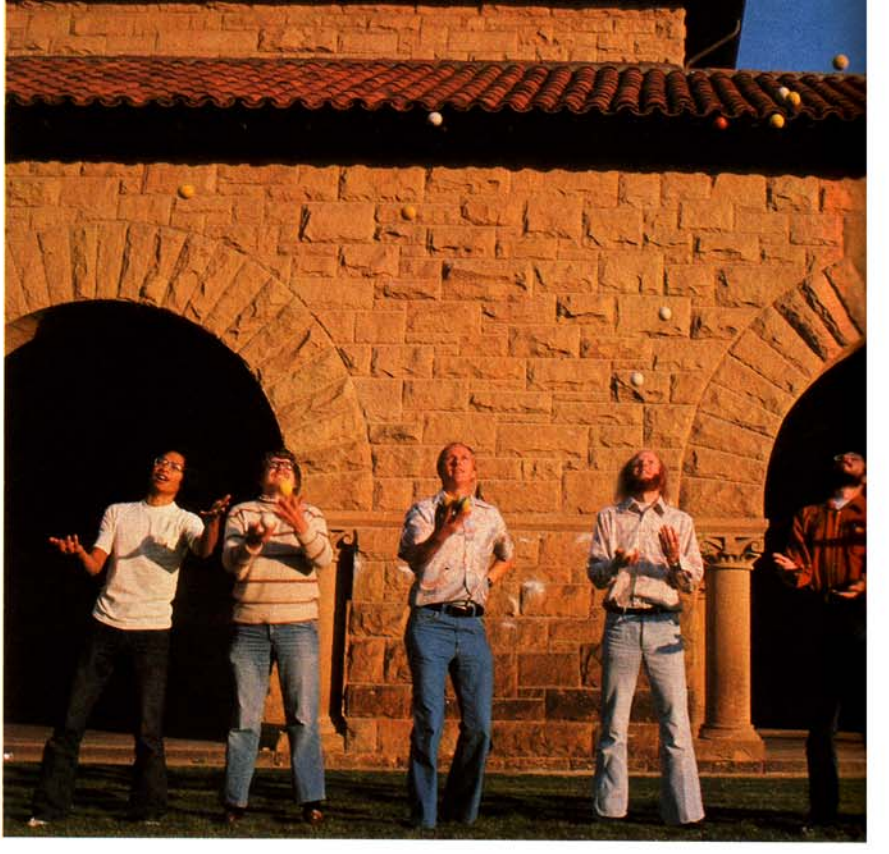
# Ronald L. Graham

By Gina Bari Kolata

**This versatile research mathematician not only finds time for problem solving, music, and sports, but also for aiding and inspiring both students and colleagues**

It is lunchtime on the sunny California campus of Stanford University, and a crowd of mathematicians, other scientists, and students gathers around mathematician Ronald L. Graham, who is spending the fall 1979 semester as a visiting professor in Stanford's Computer Science Department. He is about to give a lesson in a subject he considers one of his specialties – juggling. Graham confidently tosses several colorful balls into the air and keeps them going with the ease of an expert, which he is. At one time, he served as president of the International Jugglers Association, and he estimates he has taught about 1,000 persons to juggle.

Graham regards learning the skills of juggling and his other hobbies – gymnastics, music, trampoline, tennis, and table tennis – as being similar to learning mathematics. "The whole philosophy of learning a physical or mental skill is the same," he says. "You break a complicated skill – whether it is juggling or calculus – into much simpler parts. Then you work on the parts. Finally, after you have mastered the parts, you put the parts together. For example, when



With one hand behind his back, Graham evens the odds in a juggling contest with students at Stanford.

you juggle, you start with one ball. You work on the skill of throwing and catching that ball. When you've learned to throw and catch it accurately, you add a second ball. With mathematics, it's the same thing. Before you can learn to solve calculus problems, you must build your mathematical skills step by step."

Graham's career is devoted to mathematical research. His current job is head of the Discrete Mathematics Department of Bell Laboratories at Murray Hill, N.J. He also is an elected member of the Executive Committee of the American Mathematical Society, secretary of the mathematics section of the American Association for the Advancement of Science, and he publishes about 15 scientific papers each year. In 1972, Graham won the prestigious Polya Prize awarded by the Society for Industrial and Applied Mathematics. (George Polya, a Hungarian-born mathematician, pioneered combinatorics, the kind of math used in computer science.) Graham's colleague, mathematician Persi Diaconis of Stanford University, says that "people in his field speak of Ron Graham with a special awe."

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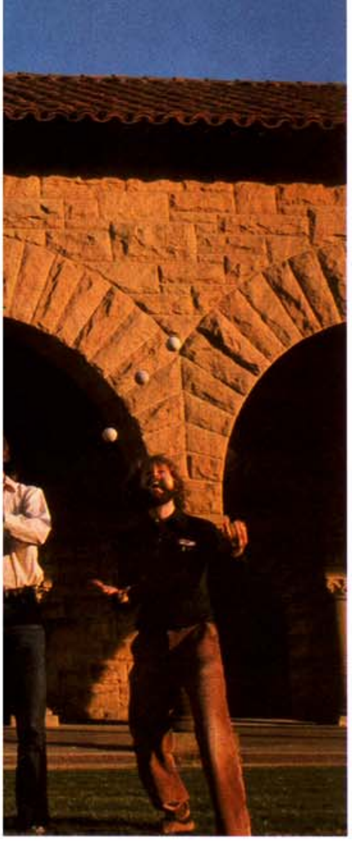
As a boy, Graham dreamed of becoming an astronomer. But he says his love of mathematics also developed very early. He enjoyed arithmetic in elementary school. While in the fifth grade, he learned how to find square roots of numbers, then tried to use the basic idea of the square-root method to find cube roots. Although it did not work, this small failure did not discourage him from setting out on a lifelong exploration of the world of math.



Graham took a beginning algebra course in the seventh grade, but soon became bored because he could do all the problems. His teacher gave him a book about calculus and introduced him to differential equations. From then on, his involvement with math deepened.

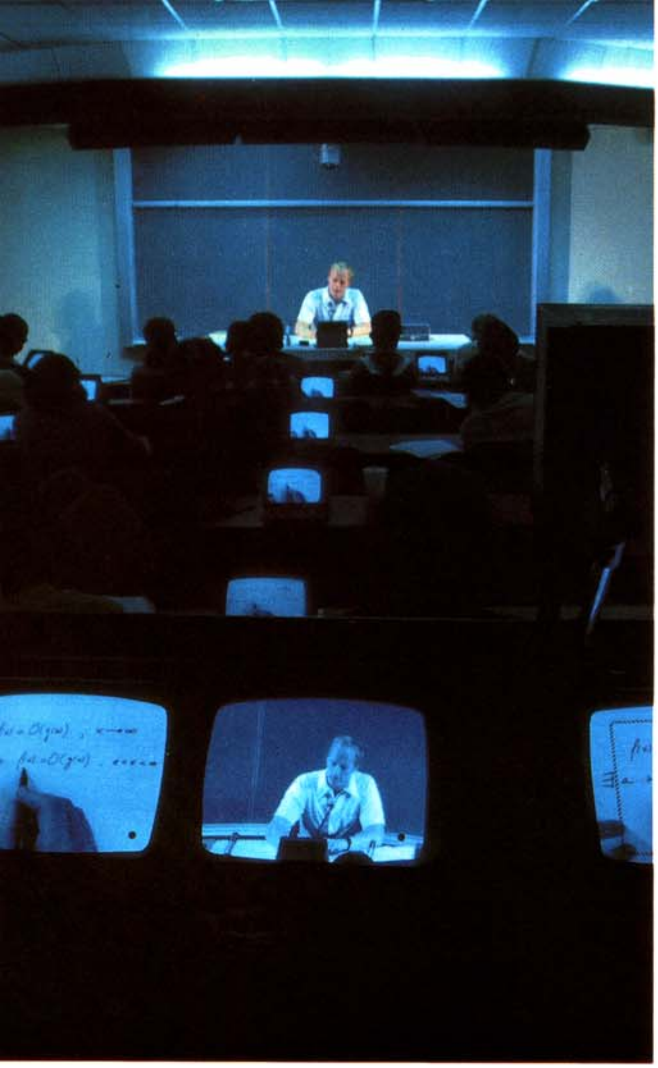
Ron Graham was born on Halloween, 1935, in Taft, Calif., a town about 241 kilometers (150 miles) northwest of Los Angeles. Graham's father worked in the oil fields near Taft until Graham was 6 years old. Then the family moved to Savannah, Ga., where his father worked in the shipyards for a short time before the family moved back to California. From then on, they shuttled between Georgia and California as his father kept changing jobs. After reaching school age, Graham never lived in any town for more than 1½ years. Eventually, his father joined the United States Merchant Marine and his parents were divorced. His mother moved to Florida, taking along with her Graham and his younger brother and sister.

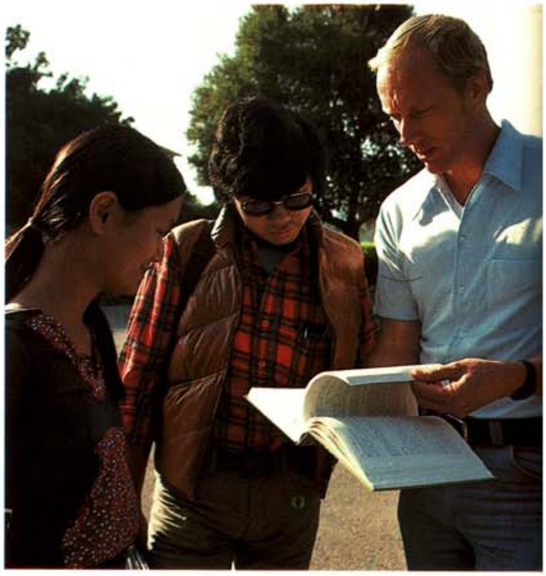
Despite this unsettled life, Graham worked hard in school and did well. He did so well on achievement tests in high school that he won a Ford Foundation scholarship. It entitled him to enter any one of four universities that had special programs allowing gifted youngsters to enroll without graduating from high school. He chose to attend the University of Chicago.

To be 15 years old and a student at the University of Chicago was both stimulating and humbling, Graham says. "Normally, each young person who entered college at such an early age was considered the best student in their little town. But when all those kids were put together, suddenly some had to face the fact that they were not the best in this college grouping."

But college gave Graham his first real opportunity to make friends and enjoy sports. He was small for his age and had also skipped grades in school. "Because we moved so often," he says, "I was always the new kid at school and – even though I'm now 6 feet 2 inches – the littlest kid." So he never was asked to participate in team sports. At the University of Chicago, however, he discovered gymnastics and juggling, sports in which being small is not a drawback. For the first time, he was accepted and respected for his physical abilities.

Graham entered the University of Chicago in 1951, with the idea of becoming a mathematician. At that time, the university admitted talented students up to two years before they were to graduate from high school and gave them degrees as soon as they completed a program that entailed reading original works in the sciences and the humanities. "That meant reading Newton for physics and Darwin for biology," Graham says. The program was designed to provide a well-rounded education. As a result, even though he was a science major, Graham took no mathematics and very few science courses because he had placed so high in his science and mathematics achievement tests. Instead, he concentrated mainly on humanities, an area in which he did not do as well on the tests.





Graham the teacher broadcasts a math course, *above,* over closed-circuit television. With his teaching assistants, *top right,* Graham tackles the paperwork of final exams. Later he pauses to answer students' questions, *above right.*

After three years in this program, Graham faced a turning point in his academic career. He ran into financial problems because his scholarship had expired. His father offered to help him financially, but only if he transferred to the University of California, Berkeley. And so Graham returned to California.

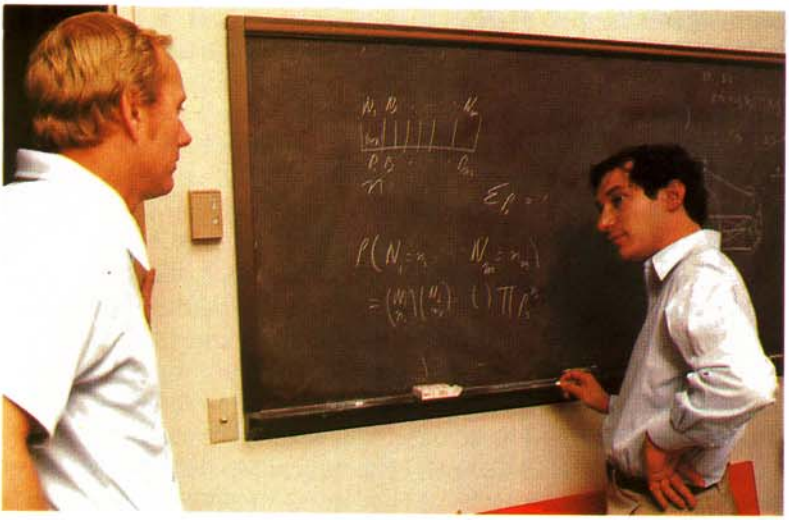
His college adviser said it would be easier for him to be accepted by the university's electrical engineering department than its mathematics department because he had no math course credits from Chicago. So he became an electrical engineering major. In the more traditional program at Berkeley, he had to take the required elementary science and engineering courses before getting into advanced studies. "After spending one year at Berkeley, I had completed four years of college," he says, "but I was not even close to a degree. And in those days, after four years of college without getting a degree, you became eligible for the military draft."

Rather than waiting to be drafted into the Army, he enlisted in the U.S. Air Force in 1955 because it promised better benefits. He was trained as a communications specialist. "This actually meant typing and sending messages," says Graham. He understood that the best student in his communications class of 100 was to be given his choice of assignments, so Graham worked like a fiend and was soon the star of the class. But he discovered that being best was not what really counted. "Those who got the good assignments," he says, "were friends with the people who made the assignments." To his dismay, Graham was sent to Alaska.

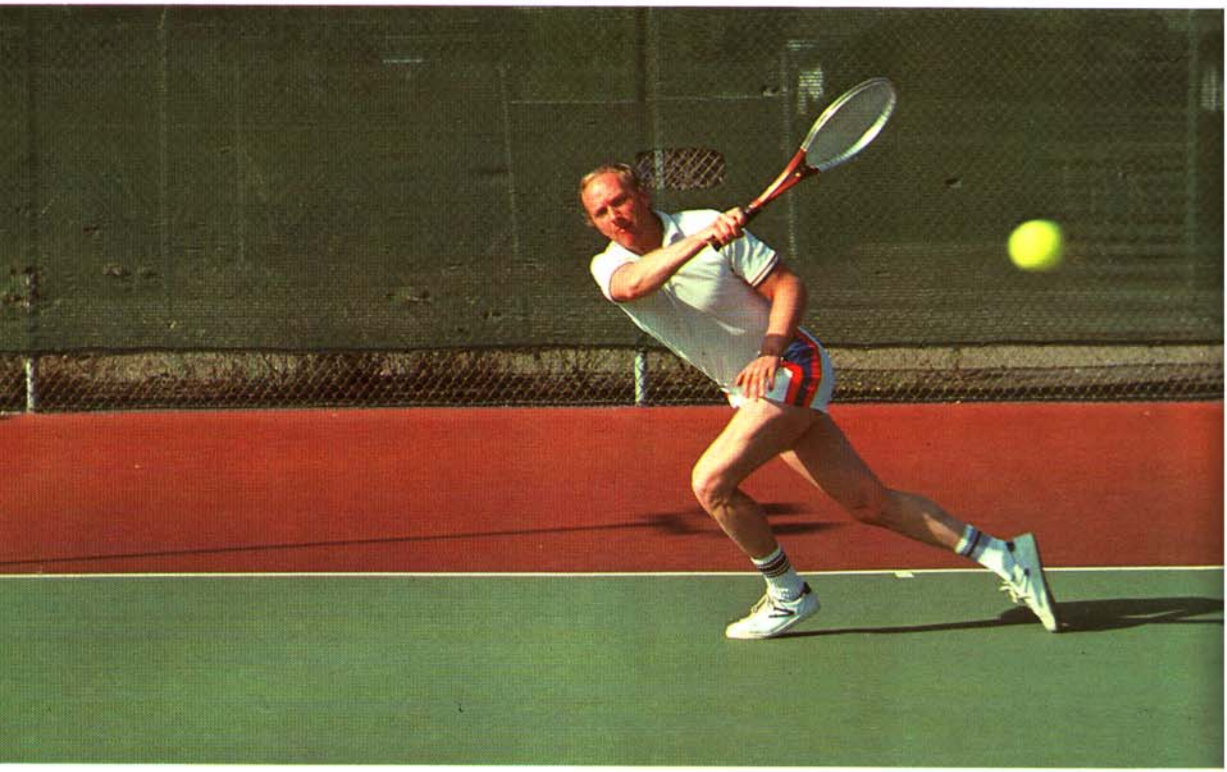
He was stationed at a large Air Force base about 42 kilometers (26 miles) from Fairbanks. Life there is bleak, especially during the winter. Fortunately, Graham obtained permission to attend the University of Alaska, about 52 kilometers (32 miles) away. "School was a real outlet," he says. He signed up to work from 7 P.M. to midnight so he could attend school during the day. Graham commuted between the base and the university by bus.

But Graham still could not major in mathematics. The university was not accredited to award math degrees. It offered only basic mathematics courses to students majoring in other disciplines. So Graham chose physics and, finally, seven years after he began college, he received a B.S. degree in 1958.

He left the Air Force in 1959, at the age of 23, and then life began to run more smoothly. Graham returned to the University of California, Berkeley, as a graduate mathematics student. There he earned his spending money as a professional trampolinist, a member of a group that performed in a circus, at supermarket openings, and in schools. At Berkeley, he met and married a classmate, math major Nancy Young. In 1962, Graham received a Ph.D. degree from the University of California and accepted a job at Bell Laboratories. He and Nancy moved to New Jersey. They had two children – a girl, Cheryl, who is now 18, and a boy, Marc, who is now 13.



Graham and a Stanford colleague, Persi Diaconis, ponder a problem in advanced mathematics.

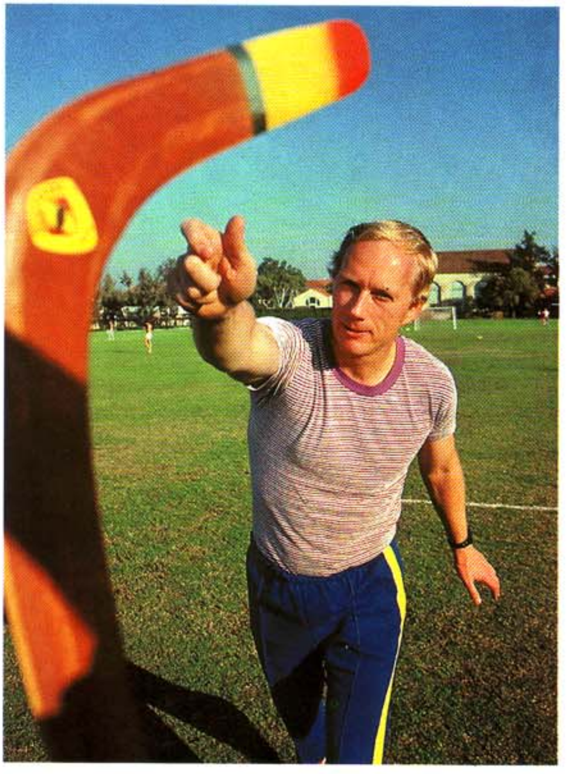


Graham the athlete slams a return across the tennis court, *above.* He also applies his finely tuned reflexes to more exotic sports, ranging from juggling balls in his Bell Labs office, *opposite page left,* to throwing a boomerang on an athletic field, *opposite page right.*

Marc, like Graham when he was Marc's age, says he wants to be an astronomer. "Math's OK," he concedes, but he does not consider it his main interest. Neither of his parents ever indicated that mathematics might be difficult, and Marc never found it so.

Cheryl's interests are more artistic than scientific. She wants to be a writer for radio or television. Nonetheless, she had no trouble learning mathematics and is currently taking calculus, the most advanced mathematics course taught in her high school. Her father always helped and encouraged her to learn mathematics, but he never pushed her to become a mathematician. He also encouraged her to learn other skills, and in 1974 she won the trampoline competition in the New Jersey State Championships. Cheryl says her father's greatest influence on her life has been in teaching her self-discipline, partly by his own example.

Cheryl and Marc had the advantage of learning about the enormous scope and power of mathematics from their father. When Marc was 10, his father showed him how mathematics can simplify a difficult problem. The problem involved trying to cover a chessboard with dominoes. The chessboard is made up of 32 black squares alternating with 32 white squares, and each square measures 1 by 1 inch. Each domino is 2 inches by 1 inch. It is easy to see that you can cover the board exactly by lining up 32 of these dominoes in 4



rows, with each row containing 8 dominoes. Now, suppose you cut one square out of the upper left-hand corner of the board and one square out of the lower right-hand corner of the board. Can you still cover the board exactly with 31 dominoes?

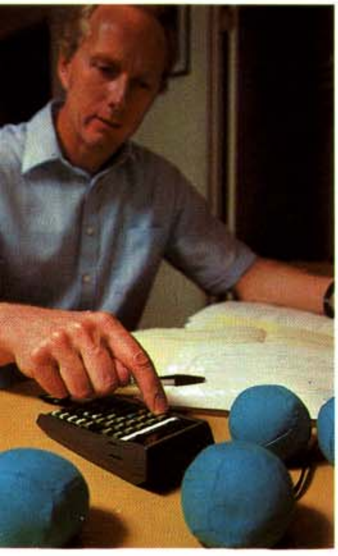
"You might try to find out," Graham says, "by physically arranging the dominoes in all possible positions on the board. But this is a long process because there are so many possible combinations of dominoes, and you are never sure if you've really tried them all."

Graham explains that it is easy to solve the problem by using a type of mathematical logic. The square you removed from the upper left-hand corner of the board is the same color as the one you removed from the lower right-hand corner. If these two squares are white, you are left with 32 black squares and 30 white squares on the board. Each domino must cover one black square and one white square, simply because that is the nature of the chessboard pattern. By breaking the problem down to these elements, the answer becomes obvious – it cannot be done. It is impossible to cover 32 black and 30 white squares with 31 dominoes, because each domino must cover one black and one white square.

The mathematics most people are exposed to in high school, or even in college, is really just the tip of the iceberg, according to Graham. *Mathematics* is a catchall term for an enormous number of disciplines. Some mathematicians work with computers. Others study the logical foundations of mathematics and never work with numbers. Mathematics covers such a broad area that, very often, mathematicians with different specialties cannot communicate with one another. They do not understand the problems the others are working on.

But some common quality makes all of these fields mathematics. "The essence of mathematics is the search for order and structure," explains Graham. "Mathematicians can deal with numbers, or with geometrical figures, or even with concepts. What unites them is their search for patterns."

Graham's work often focuses on problems that arise at the boundary between mathematics and computer science. For example, he looks for solutions to a group of problems called "hard" problems. These problems are generally easy to state and occur in countless practical situations. But the only known ways guaranteed to solve most of these problems would involve so many trillions of calculations that they would require centuries of computer time. However, researchers have shown that if they could find a short cut to solving one hard problem, they could adapt that short cut to solve them all.

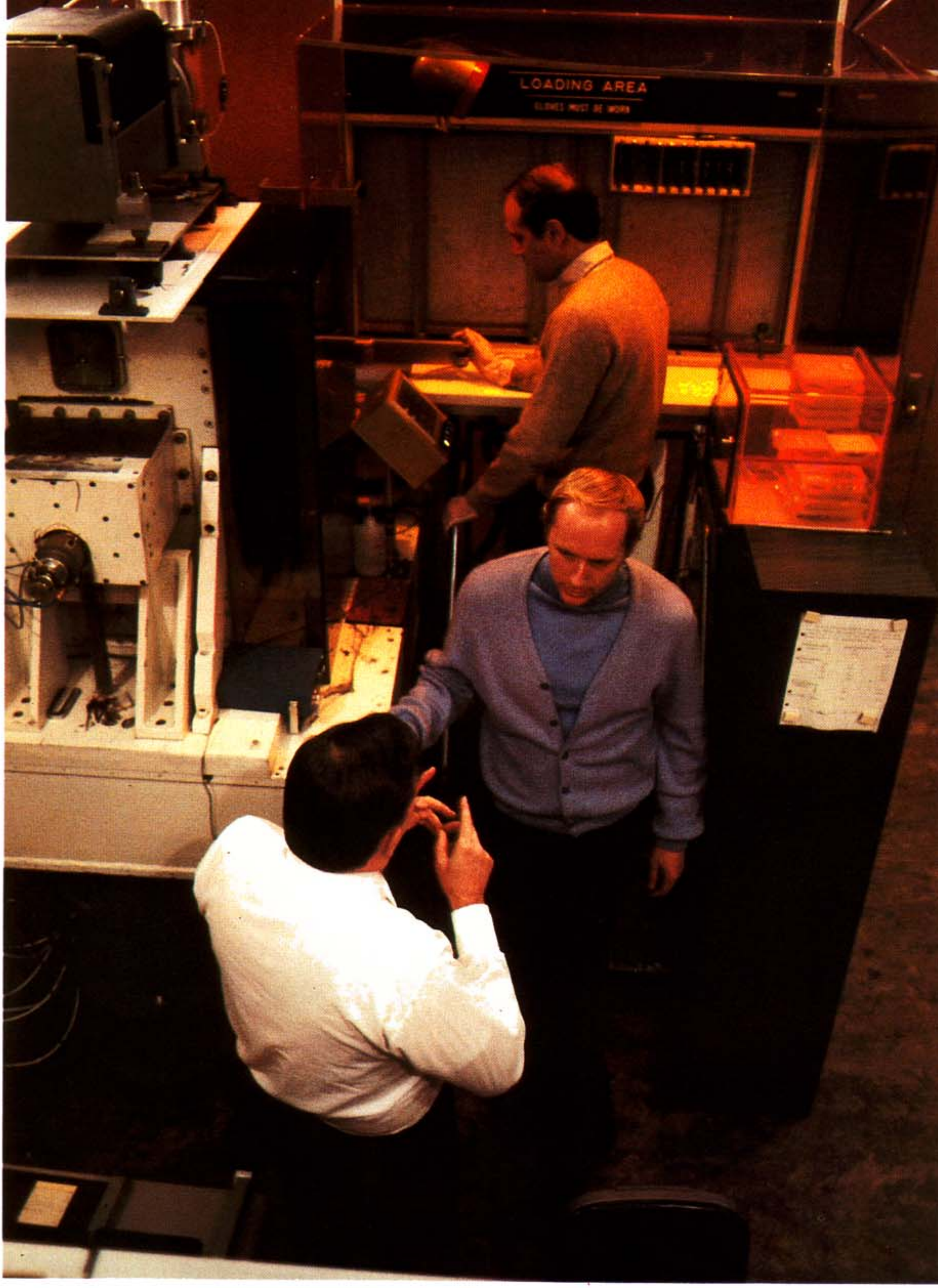


Graham the researcher checks some figures in his office, *top,* then gets together with his staff, *above,* to discuss their projects.

One of the best-known examples of hard problems is the traveling salesman problem – find the shortest tour of a group of cities in which each city is visited only once. This problem turns up in a number of everyday situations. Telephone companies, for example, must solve a traveling salesman problem to plan collections from the coin boxes of pay telephones. Yet there is no easy way to solve this kind of problem short of measuring the distances of all possible routes between cities and then comparing all of the possible combinations of these routes to find the shortest combination. For a tour of only 60 cities, for example, it would take a computer billions of centuries to test all of the possible route combinations.

Another hard problem involves objects of different weights that must be divided into two or more groups so that the groups are as equal in weight as possible. Graham explains that this becomes a scheduling problem if you substitute the word "time" for "weight." How can you assign tasks, each of which takes a certain length of time, to several workers so that the workers complete all the tasks in the shortest possible time? Once again, the only known way to solve this problem is to try all the possibilities. Again, this can take an enormous amount of computer time when there are many tasks.

Graham devised a method that allows mathematicians at least to begin dealing with hard problems. His method is called a "worst-case analysis" because it tells what happens in the worst of all circumstances using some particular technique. This is now a major thrust in research on solutions to hard problems. Graham first applied his worst-case analysis to a weighing problem. Suppose you want to divide a group of objects into two piles being as equal in weight as possible. One way to do this would be to first arrange the weights in




Graham swaps ideas with a Bell Labs engineer on a problem involving an electron-beam device used in making integrated circuits.

Graham shares a playful moment with his son, Marc, and daughter, Cheryl, on the sundeck of his New Jersey home.

decreasing order. Then put the heaviest weight in the first pile and the next heaviest weight in the second pile. Thereafter, add each weight to the pile that weighs the least at that point.

If you follow this grouping pattern with five objects weighing, for example, 3, 3, 2, 2, and 2 pounds, one pile will consist of objects weighing 3, 2, and 2 pounds; the other pile, 3 and 2 pounds. The heaviest pile, then, has a total weight of 7 pounds; the lightest, 5 pounds. This is the worst solution this method will produce. In this simple case, it is easy to find the best solution – one pile of objects weighing 3 and 3 pounds and the other of objects weighing 2, 2, and 2 pounds. Note that the heaviest pile of the best solution weighs 6 pounds and that of Graham's solution weighs 7 pounds; the ratio between the two is 7 to 6. Graham has shown that no matter how complicated such a problem is, this ratio will hold. Worst-case analysis is important in such complicated problems as scheduling tasks and work shifts for thousands of employees in a factory.

Graham shared the 1972 Polya Prize for his work in a branch of mathematics called Ramsey theory. The theory was named after Frank P. Ramsey, brother of the former archbishop of Canterbury. Before his death at age 26, Ramsey began to develop the mathematical branch that bears his name. "The basic philosophy of Ramsey theory," Graham says, "is that complete randomness is impossible. There is always structure somewhere." For example, a simple result in Ramsey theory states that the numbers 1 through 101 arranged in any order will always contain, interspersed among the entire 101 numbers, 11 numbers which are in increasing order or 11 numbers which are in decreasing order.

Another example of Ramsey theory deals with relationships among pairs of objects, ranging from dots on paper to people in a crowd. Any two dots can either be connected by a line or not connected. Any two persons can either know each other or be strangers. Ramsey theory tells us that, in any collection of six dots, we are guaranteed of always finding either three dots with all pairs connected or three dots with none of the pairs connected. Likewise, Ramsey theory states that in any randomly selected group of six people, there will always be either three people who all know each other or three people who are complete strangers.

According to Graham, Ramsey theory problems get very hard very fast. For example, to guarantee that a group of people will contain four who either all know one another or are all complete strangers, the group must consist of at least 18 persons. But no one has any idea of exactly how many people must be in a randomly selected group to guarantee that at least five of these people will either all know one another or be strangers.

Graham shared the Polya Prize for work on Ramsey theory with Bruce L. Rothschild of the University of California, Los Angeles, and Klaus Leeb of the University of Erlangen in West Germany. In a joint paper, these three mathematicians showed that a much larger collection of mathematical structures have Ramsey properties than anyone had suspected. Ramsey theory is not very useful in solving

Cheryl and Marc receive a lesson from their father in the dangerous but graceful art of trampoline gymnastics.

In a quiet moment alone, Graham applies to music the same powers of learning and concentration that make him a proficient athlete and a great mathematician.

practical problems, but Graham believes that the new ways in which he learns to think in the process of solving Ramsey-type problems help him in solving practical problems.

At Bell Laboratories, Graham works on practical problems that have stumped researchers in other areas of the laboratories. For example, he has worked on problems involving the creation of more intense laser beams. This work is important because telephone companies are beginning to use systems for transmitting voice and data signals on beams of light through thin glass fibers, rather than on electrical impulses through copper wire. Lasers are used to generate the light beams at the transmission ends of the fibers and boost, or strengthen, them at regular intervals as they travel along the fibers. With a more intense laser beam, the light will travel farther through a fiber before it needs to be boosted.

Graham's unusual college experience has proved to be a real asset in his position at Bell Laboratories. Henry O. Pollak, director of the Mathematics and Statistics Research Center, believes that Graham's undergraduate training in physics and engineering helps him communicate with engineers and systems developers and planners. Mathematicians and engineers use different kinds of jargon, or specialized terms, so they often find it difficult to communicate even when they are talking about the same things. "Ron is bilingual in the languages of mathematics and engineering," says Pollak.

Although people often think of mathematicians as isolated thinkers, doing little besides working alone in tiny rooms with their pencils and pads of paper, this is not the case. Like many mathematicians, Graham collaborates a great deal with other mathematicians and scientists. A clear and articulate speaker, he gives about 20 talks a year. He also attends 10 to 15 scientific meetings a year and enjoys traveling and socializing with other researchers. "I find new places and new people stimulating," he says. "With personal contact, you can find out what the current problems are. If you wait and read the journals, you will be two years behind."

A typical day for Graham at Bell Laboratories is hectic and usually leaves him little time for his own research. He must constantly attend to various letters and telephone calls. Often his correspondence concerns his duties as a member of the editorial boards of 21 scientific publications. He consults with people in his department on problems they encounter in their work. He also spends time with scientists and engineers from other departments at Bell Laboratories who have problems they hope Graham and his group can solve. So Graham does most of his research at night or on airplanes when he travels, because this mathematical research requires quiet, uninterrupted stretches of time.

Graham finds the creative aspects of mathematics research truly exciting. Even when he is not consciously wrestling with a problem, a part of his brain is working on it. Often the solution comes to him out of the blue, when he least expects it. "It's almost as though your brain has a life of its own," he says. "You think about a difficult problem during the day, go to sleep, then wake up the next morning and things are much clearer. It's as though sleep gives the brain a chance to reorganize the information that has come in."

Diaconis was quite impressed with the amount of work that Graham managed to produce at Stanford University. "I asked him a question one day," Diaconis says. "Soon he returned with a large folder containing 40 or 50 pages of calculations."

Calculations aid Graham in searching for structures. He breaks down a general problem, then looks over his calculations to see whether they have something in common, some pattern that keeps recurring. When he sees the patterns emerge, complex problems begin to fall into place and become more manageable.

In order to detect these often subtle patterns, Graham stresses that it is crucial to keep an open mind – to be willing to look at problems in unusual ways. "I think it is important to try to keep your brain flexible," he says. To do this, he constantly learns new skills. For example, he studied Chinese for two years and then turned to learning to play the piano. He even bought a portable electric piano so he can practice at least one hour each day even on trips.

His list of skills seems endless. Besides music, juggling, trampoline, and gymnastics, he is fond of bowling, jogging, tennis, table tennis, throwing a boomerang, running in marathon races, and skating. Learning all these skills, Graham says, reflects his desire to keep using different parts of his brain. "It just makes you look at things from a different point of view. Things are moving so fast that much of the knowledge you have now will probably be obsolete in 10 years. But if you keep yourself familiar with how to break down complicated tasks into simpler skills and then practice putting those skills together, you can continue to learn," Graham explains.

Graham is greatly disturbed by evidence that students today may not be learning to learn. In particular, he mentions a recent survey showing that the ability of high school students to solve simple mathematical problems is decreasing. He believes that students who fail to learn basic mathematics are permanently handicapped in their ability to function in the world. "As I look around, I see mathematics everywhere," he says. "Take juggling, for example. Some patterns are possible only with an even number of balls, some only with an odd number of balls. Why? Because we only have two hands. Jugglers usually divide an even number of balls between their two hands, throwing and catching half with one hand and half with the other. When they work with an odd number of balls, they usually throw all the balls from one hand to the other. If we had three hands," he says, "we would use entirely different patterns."

The fact that most students dislike math is nothing new, but Graham suspects that current low test scores may be related to the advent of the new math in the 1960s. This experiment in teaching mathematics to elementary and high school students was designed by educators who believed that mathematics, as it was then taught, was old-fashioned. All the sciences were developing and changing, and so should mathematics. Students in physics classes learned about discoveries in quantum mechanics and relativity. Biology students studied genes and deoxyribonucleic acid (DNA). But mathematics students were taught as though nothing new had happened since Sir Isaac Newton developed calculus 300 years before.

To modernize mathematics, educators decided to introduce some abstract 20th-century concepts, such as set theory. These concepts, however, were foreign to many teachers. Graham believes that all too often the teachers communicated their dislike, distrust, and misunderstanding of the new math to their students. Parents also were confused by the new math, and many were afraid even to try to help their children with their homework. As a result, many students felt alienated by their math courses, failing to see how mathematics could apply to their lives.

In addition, Graham explains, the new math was often dull. "It did not lend itself to interesting exercises and examples. It wasn't a very good playground for students to have fun."

But, according to Graham, even students not exposed to the new math have felt put off by mathematics because most teachers do not feel much enthusiasm for any kind of math. "Teachers often take just a few math courses and look upon mathematics as a chore," Graham says. "For me, mathematics is exciting and alive, and there's always something new around the corner." Graham believes it should be possible to teach mathematics in a way that is more fun at the elementary levels. Students could analyze games or work with mathematical models of real-life situations. His idea is not to turn everyone into mathematicians – he simply wants everyone to be comfortable with math. "Mathematics is at the base of almost all technical areas," he says. "If you don't feel comfortable with mathematics, these areas will always be difficult."

Graham makes his own efforts to show students the excitement of mathematics. He gives talks to high school students and writes popular articles about various new developments in the field to dispel the popular impression that "mathematics is dead and buried in textbooks." In his talks, he shows how mathematics applies to many problems in the world, from creating or breaking secret codes to developing computer programs and even to scheduling classes or homework efficiently. He finds that most of the students he talks with are amazed that mathematics can be so lively.

With all of his activities, Graham's colleagues continually ask themselves how he finds time to do any research. "I think he works 24-hour shifts," remarks Fan Chung, a member of Graham's research group at Bell Laboratories.

Yet Graham never seems harried. He always acts as though he has all the time in the world for each person who wants to talk with him. Chung believes that she is a much better mathematician for having worked with Graham. He encouraged her to try more challenging problems than she had ever approached before, and he helped her develop a feeling for the kinds of problems that might be of the most interest to her. "He makes people feel free to go to him for help," she says. "Ron guided me into a new level of mathematics, and I'm not the only one for whom he did this."

Pollak agrees. "One of Ron's responsibilities at Bell Laboratories is to guide people in interesting directions. He is very, very good at this. He is willing to take the time and he is good at working with people. He is full of ideas and doesn't mind sharing them."

Graham says that, out of necessity, he developed the knack of changing gears quickly. "If you let yourself think that there's so much to do that you can never do it all, you might get bogged down and wind up not doing anything," he says.

Nevertheless, mathematics research for him is not an on-again off-again process. "Part of your brain is always running," he says. So, as Graham the juggler performs before the crowd at Stanford University, or Graham the pianist practices music, it is highly likely that, simultaneously, Graham the mathematician is solving problems in some corner of his mind.