3↑↑↑3 etc., etc.

Graham's number

The World Champion largest number, listed in the latest *Guinness Book of Records*, is an upper bound, derived by R. L. Graham, from a problem in a part of combinatorics called Ramsey theory.

Graham's number cannot be expressed using the conventional notation of powers, and powers of powers. If all the material in the universe were turned into pen and ink it would not be enough to write the number down. Consequently, this special notation, devised by Donald Knuth, is necessary.

3↑3 means '3 cubed', as it often does in computer printouts.

3↑↑3 means 3↑(3↑3), or 3↑27, which is already quite large: 3↑27 = 7,625,597,484,987, but is still easily written, especially as a tower of 3 numbers: 3^{3^3}.

3↑↑↑3 = 3↑↑(3↑↑3), however, is 3↑↑7,625,597,484,987 = 3↑(7,625,597,484,987↑7,625,597,484,987), which makes a tower of exponents 7,625,597,484,987 layers high.

3↑↑↑↑3 = 3↑↑↑(3↑↑↑3), of course. Even the tower of exponents is now unimaginably large in our usual notation, but Graham's number only starts here.

Consider the number 3↑↑↑ . . . ↑↑↑3 in which there are 3↑↑↑↑3 arrows. A largish number!

Next construct the number 3↑↑↑ . . . ↑↑↑3 where the number of arrows is the previous 3↑↑↑ . . . ↑↑↑3 number.

An incredible, ungraspable number! Yet we are only two steps away from the original ginormous 3↑↑↑↑3. Now continue this process, making the number of arrows in 3↑↑↑ . . . ↑↑↑3 equal to the number at the previous step, until you are 63 steps, yes, *sixty-three*, steps from 3↑↑↑↑3. That is Graham's number.

There is a twist in the tail of this true fairy story. Remember that Graham's number is an upper bound, just like Skewes' number. What is likely to be the actual answer to Graham's problem? Gardner quotes the opinions of the experts in Ramsey theory, who suspect that the answer is: 6 !!

'Mathematical Games', *Scientific American*, November 1977.