JOURNAL OF COMBINATORIAL THEORY 12 (B), 203–204 (1972)

Note

A Simpler Counterexample to the Reconstruction Conjecture for Denumerable Graphs

J. FISHER,[1] R. L. GRAHAM,[2] AND F. HARARY[3]

Received August 2, 1971

In the note [1], one of us constructs two non-isomorphic countably infinite graphs G and G' which serve as a counterexample to the question in [2], in which another of us asked whether the Reconstruction Conjecture is true for infinite graphs. The object of this note is to present two simpler graphs G and G' which serve the same purpose.

The first graph G is constructed as follows, and is point-symmetric. Start with a single point v_0 in the plane and let it be adjacent to a denumerable number of other points $v_1, v_2, v_3, \ldots$. Let each of these points v_i then be adjacent not only to v_0 but also to a denumerable number of other points $v_{i1}, v_{i2}, v_{i3}, \ldots$. Continue this process indefinitely.

For the second graph G', not isomorphic to G, we simply take the disconnected denumerable graph consisting of two copies of G. In the notation of [3, p. 21], we have $G' = 2G$. Then on the removal of an arbitrary point of either G or G' all of the resulting subgraphs are isomorphic and consist of a denumerable number of copies of G.

We mention in conclusion that we have here a denumerable number of non-isomorphic graphs, namely, $G, G' = 2G, 3G, \ldots; \aleph_0 G$, all of whose single point deleted subgraphs are isomorphic! Note that we do not have here two infinite *trees* as our counterexample and do not know whether such an example exists. For locally finite infinite trees, we think that the Reconstructive Conjecture will hold.

[1] Courant Institute of Mathematical Sciences, New York University, New York, NY 10012.

[2] Bell Laboratories, Murray Hill, NJ 07974.

[3] Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48104.

References

1. J. Fisher, A counterexample to the countable version of a conjecture of Ulam, *J. Combinatorial Theory* 7 (1969), 364–365.
2. F. Harary, On the reconstruction of a graph from a collection of subgraphs, "Theory of Graphs and Its Applications" (M. Fiedler, ed.), Academic Press, New York, 1965, pp. 47–52.
3. F. Harary, "Graph Theory," Addison-Wesley, Reading, Mass., 1969.

Printed in Belgium