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ON SWITCHING NETWORKS AND BLOCK DESIGNS

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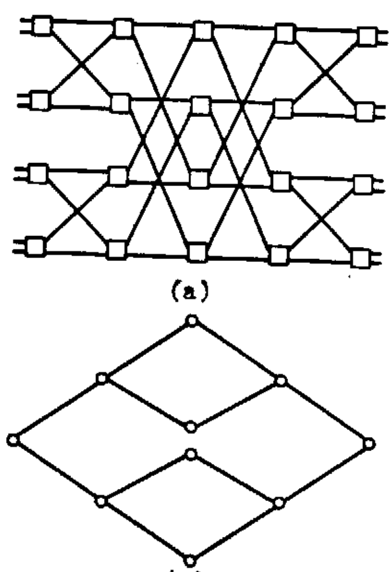
ABSTRACT

A switching network can be viewed as a collection of interconnected crosspoints which provides connections between input terminals and output terminals. The linking pattern of a switching network refers to the scheme by which the crosspoints are interconnected. Two switching networks having the same number of crosspoints but with different linking patterns can in general perform quite differently. In this note, we briefly introduce some measurements of the blocking performance of switching networks. We also summarize some of the main results in this area and give a new technique for the construction of large classes of interesting switching networks. The construction will depend on the use of certain combinational structures, called block designs, for determining the linking patterns of the networks.

1. INTRODUCTION

We consider a multi-stage network composed of rectangular switches, a set of links interconnecting the switches, and two sets of terminals, namely, the set of input terminals I and the set of output terminals Ω. For an input terminal u and an output terminal v, the linear graph for u and v, denoted by G(u,v), is defined to be the union of all paths that can be used to connect u and v (see Fig. 1). A network is said to be balanced if all the linear graphs G(u,v), $u \in I$, $v \in \Omega$, are isomorphic[1,7].

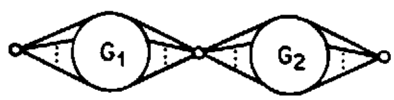


(a)

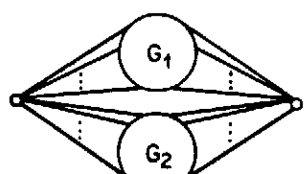
(b)

Figure 1. A multi-stage network and its linear graph. Note that the switches in the network in (a) become nodes of the linear graph in (b).

and it is symmetric with respect to the two middle stages.



(a) SERIES COMBINATION



(b) PARALLEL COMBINATION

Figure 3. The structure of a series-parallel linear graph.

Comparisons in the literature have been made involving the following classes of linear graphs:

(i) The class of series-parallel linear graphs with the same transparency. Roughly speaking, the more regular a linear graph is, the better it is.

(ii) The class of symmetric regular series-parallel linear graphs. A graph with degree sequence $(\lambda_1,\lambda_2,\ldots,\lambda_{\lfloor t/2\rfloor})$ is better than another graph with degree sequence $(\lambda'_1,\lambda'_2,\ldots,\lambda'_{\lfloor t/2\rfloor})$ if and only if $\prod_{i=1}^{j} \lambda_i \geq \prod_{i=1}^{j} \lambda'_i$ for any j, $1 \leq j \leq \lfloor t/2\rfloor$. Roughly speaking, the more "spread-out" a linear graph is, the better it is.

For details and proofs, the reader is referred to Chung and Hwang[2,3] (this includes other results of this type, as well).

3. BLOCK DESIGNS

Before we give construction for zone-balanced networks, we will first introduce the idea of a block design.

A (b,v,r,k,λ)-block design is a family of subsets $X_1,X_2,\ldots,X_b$ of a v-element set X, satisfying the following conditions:

(1) Each X_i has k elements, $1 \leq i \leq b$.

(2) Each 2-element subset of X is a subset of exactly $\lambda > 0$ of the sets $X_1,\ldots,X_b$.

Properties (3) and (4) follow immediately from (1) and (2).

(3) Each element of X is in exactly r of the sets $X_1,\ldots,X_b$.

(4) $r(k-1) = \lambda(v-1)$ and $bk = vr$.

For example, the following is a (7,7,3,3,1)-block design.

$$X_1 = \{0,1,3\}$$
$$X_i = \{i,i+1,i+3\} \pmod 7 \quad \text{for } i = 1,\ldots,6.$$

The reader is referred to Hall[6] or Collens[4] for the existence and constructions of various classes of block designs. Hagelbarger[5] first proposed in 1973 the use of block designs for constructing switching networks. In the next section we will see that block designs are very useful for designing zone balanced networks.

4. THE DESIGN OF ZONE-BALANCED NETWORKS

We will restrict ourselves for the remainder of the paper to series-parallel linear graphs which are both regular and

Many switching networks also have the property that the set of input terminals and the set of output terminals can be partitioned into a number of zones such that requests for calls connecting two terminals in the same zone are more likely than those connecting terminals in different zones. A network is said to be zone-balanced if it has two nonisomorphic linear graphs, say G_1 and G_2, so that the linear graph G(u,v) is isomorphic to G_1 if u and v are in the same zone and G(u,v) is isomorphic to G_2 if u and v are in different zones (see Fig. 2).

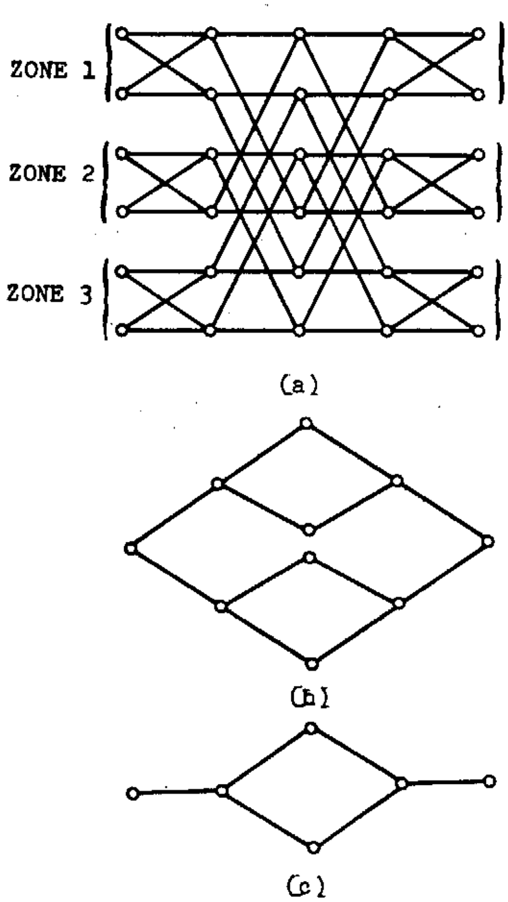


Figure 2. A zone-balanced network and its linear graphs.

One reason why balanced and zone-balanced networks are of interest in switching theory is because of the relative ease with which their theoretical performance can be evaluated.

2. PRELIMINARIES

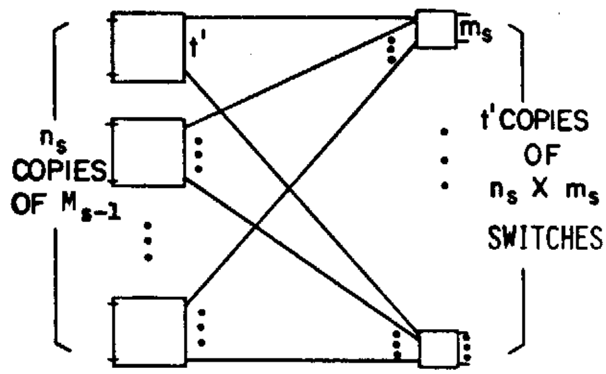
To determine the exact blocking performance of a switching network is in general a very complex and difficult problem. One of the most popular methods of doing this (called Lee's model[9]) is to calculate the blocking probabilities of the underlying linear graphs under certain independence assumptions. There is also another measure of blocking in switching networks, called transparency, which is defined to be the average number of non-blocking paths between an input terminal and an output terminal[10]. Both of these measures are reasonably correlated to the actual blocking performance of a switching network. While transparency is much more easily calculated than the Lee blocking probability, it is in general a much coarser measure. In a balanced network, transparency is proportional to the number of distinct paths in the linear graph (for a fixed traffic load). With these two measures we can then compare linear graphs and consequently, the corresponding switching networks. Before we summarize some of the main results in comparing linear graphs, we will first define the following types of linear graphs.

A linear graph is said to be series-parallel if it is either a series combination or a parallel combination of two smaller linear graphs (see Fig. 3). A linear graph is said to be regular if any two nodes in the same stage have the same indegree and outdegree (i.e. the numbers of links coming into a node and going out of a node, respectively). A linear graph is said to be symmetric if the number of stages is odd and it is symmetric with respect to the middle stage, or, the number of stages is even symmetric. Since the sets of input terminals and output terminals under consideration are of the same size, the zone-balanced network will also be symmetric with respect to the center stage.

Let the sets of input terminals I and output terminals Ω be partitioned into v zones, i.e., $I = I_1 \cup I_2 \cup \ldots \cup I_v$, $\Omega = \Omega_1 \cup \Omega_2 \cup \ldots \cup \Omega_v$ where each I_i and Ω_i has t elements. Thus, I and Ω each have tv elements. I_i and Ω_j will be considered to be in the same zone if and only if $i = j$. The linear graph $G_1 = G(u,v)$, where $u \varepsilon I_i$, $v \varepsilon \Omega_i$, is called the <u>internal linear graph</u> of the network and the graph $G_2 = G(u',v')$, where $u' \varepsilon I_i$, $v' \varepsilon \Omega_j$, $i \neq j$, is called the <u>external linear graph</u> of the network. Later in this section we will construct a zone-balanced network which has its internal linear graph with t distinct paths and its external linear graph with $\lambda\omega$ distinct paths where $t = r\omega$, provided a (v,b,r,k,λ)-block design exists.

The zone-balanced networks we construct can be divided into three parts. The primary part consists of a few stages where traffic distribution takes place within each zone so that each input terminal has sufficient access to the central stage. The secondary part is the central stage which provides interconnections between different zones. The tertiary part plays the same role for the output terminals as the primary part does for the input terminals.

The primary part of the network can be viewed as v copies of a network M_s, which is an s-stage network with switches in stage i having size $n_i \times m_i$ for $1 \le i \le s$ (see Fig. 4).



<u>Figure 4</u>. The distribution network M_s

We note that we can vary the number s of stages in M_s depending upon the sizes of I and Ω and the zone sizes in order to reduce the cost (the number of crosspoints) of the network[10].

Here is an explicit method for the interconnections in each copy of M_s. While the notation may appear at first to be somewhat complicated, in fact, it will turn out to be extremely useful in specifying the linking pattern of the network.

Let $f^j(i_1,i_2,\ldots,i_{s-1};v')$ denote the $(i_1+i_2m_1+\ldots+i_{s-1}m_1\cdots m_{s-2}+v'm_1\cdots m_{s-1})$-th switch in stage j where $0 \le i_q \le m_q$, $1 \le q \le s$, $0 \le v' < v$.

Let $f_q^j(i_1,i_2,\ldots,i_{s-1};v')$ denote the q-th outlet line of the switch $f^j(i_1,\ldots,i_{s-1};v')$ where $0 \le q < m_j$. Then we have:

$f_q^j(i_1,\ldots,i_{s-1};v')$ is connected to

$f^{j+1}(i_1,\ldots,i_{j-1},q,i_{j+1},\ldots,i_{s-1};v')$

for any $1 \le j < s$.

In Figure 5, we give **an example with** $m_1 = m_2 = m_3 = 2$.

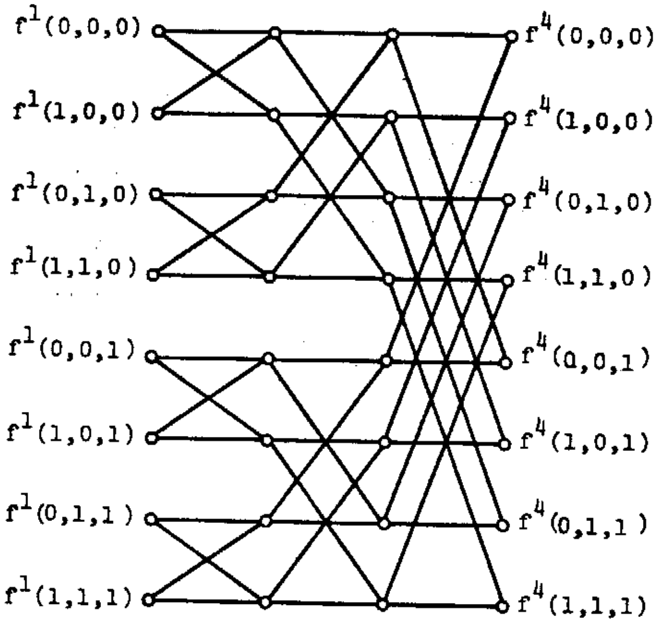
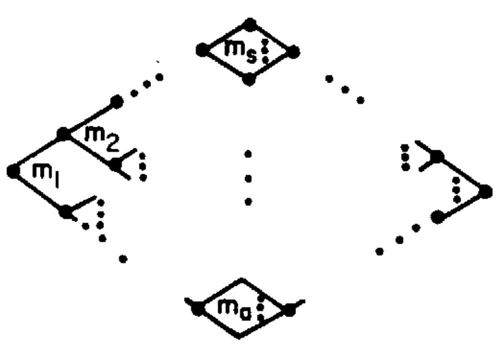
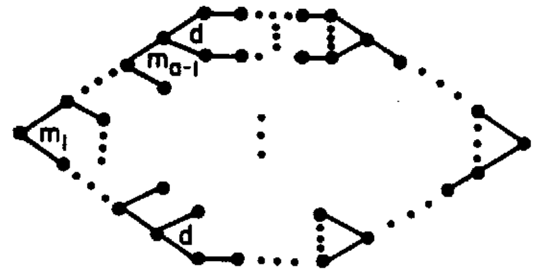


Figure 5. Labeling an example of a primary part.

The internal linear graph has $m_1 \ldots m_s$ distinct paths as shown in Figure 6(a). The external linear graph depends on ω. Based on the discussions in Section 2, the more "spread-out" the linear graph is, the better it is. First we will consider the most "spread-out" case when $\omega = m_1 \ldots m_{a-1}d$ where $a < s$ and d is equal to m_a. In the other cases, the zone-balanced network can be constructed similarly. The external linear graph is shown in Figure 6(b).



(a)
The internal linear graph



(b)
The external linear graph

Figure 6. Linear graphs of zone-balanced networks.

The central stage of the zone-balanced network consists of $b\omega$ switches of size $k \times k$. We define

$f^c(i_1,\ldots,i_a\ b')$ to be the $(i_1+i_2m_1+\ldots+i_am_1\ldots m_{a-1}+b'm_1\ldots m_{a-1}d)$-th switch

where $0 \le i_q < m_q$, $1 \le q < a$,

$0 \le i_a < d$ and $0 \le b' < b$.

Let $X_1, X_2, \ldots, X_b$ denote the sets of a (v,b,r,k,λ)-block design. For any element $y \in X = \bigcup_i X_i$, we say the i-th y-set is X_j if X_j is the i-th set containing y, i.e., $|\{X_q : y \in X_q, 1 \le q \le j\}| = i$. Now, we are ready to describe the interconnections between the primary part and the central part of the zone-balanced network.

First, we consider the special case when $d = m_a$. $f_q^s(i_1,\ldots,i_{s-1};v')$ is connected to $f^c(i_1,\ldots,i_a;b')$ if $v' \in X_{b'}$, and the $(i_{a+1}+i_{a+2}m_{a+1}+\ldots i_{s-1}m_{a+1}\ldots m_{s-2}+qm_{a+1}\ldots m_{s-1})$-th v'-set is $X_{b'}$.

Finally we connect the central part and the tertiary part in the same way (symmetrically) that the primary and central parts were connected. It is now easy to check (using the definition of a block design and the distributic properties of M_s) that the internal linear graph and the external linear graph of our network are as shown in Figure 6.

Now, if d is a proper divisor of m_a, the above scheme has to be modified slightly.

Note that i_a can be written as $i'_a + i''_a d$ where $0 \le i'_a < d$. Then we have:

$f^s_q(i_1,\ldots,i_{s-1};v')$ is connected to

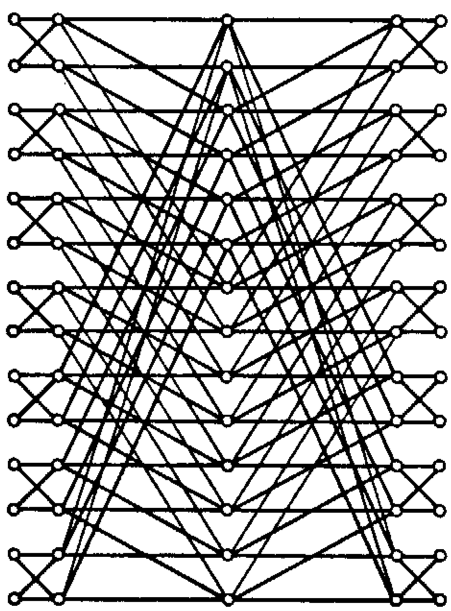
$f^c(i_1,\ldots,i_{a-1},i'_a;b')$

if $v' \varepsilon X_b$, and the

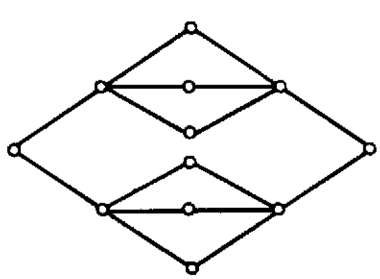
$(i'_a+i_{a+1}d+\ldots+i_{s-1}dm_{a+1}\cdots m_{s-2}+qdm_{a+1}\cdots m_{s-1})$-th v'-set

is $X_{b'}$.

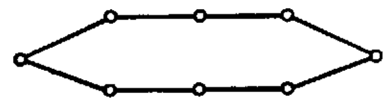
The switching network illustrated in Figure 7 is constructed from the (v,b,r,k,λ)-block design given in Section 3, i.e., $X_i = \{i,i+1,i+3\}$ (mod 7) for $i = 0,1,\ldots,6$.



(a)



(b)



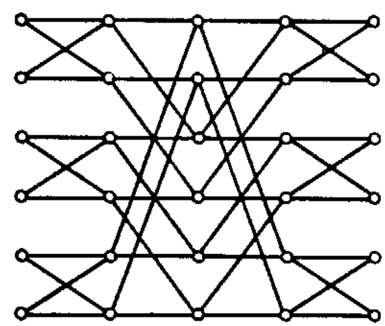
(c)

Figure 7. The zone-balanced network constructed by using the (7,7,3,3,1)-block design.

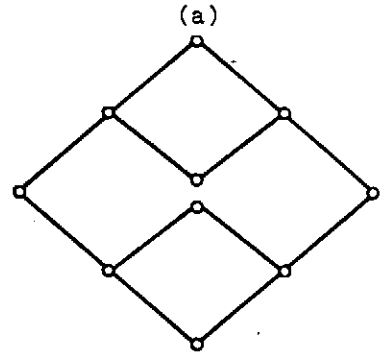
We note that by modifying the construction mentioned above, we could easily obtain zone-balanced networks with various external linear graphs (see Figure 2). However, the linking patterns we give above result in a switching network with the best linear graphs among the class of all internal and external graphs with the given number of distinct paths. For example, using the (3,3,2,2,1)-block design

$X_1 = \{1,3\}$, $X_2 = \{1,2\}$, $X_3 = \{2,3\}$,

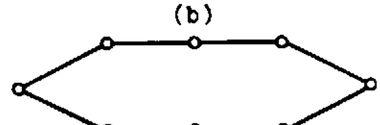
we obtain the network shown in Figure 8.



(a)



(b)



(c)

Figure 8. A zone-balanced network constructed by our scheme.

We also note that networks in Figure 2 and Figure 8 have the same number of crosspoints but the linear graph in Figure 8(c) is better than the linear graph in Figure 2(c).

5. REMARKS

The construction scheme for zone-balanced networks we presented in Section 4 can be easily extended to multi-zone-balanced networks as follows:

Suppose the set of input terminals and the set of output terminal can be partitioned into a number of zones which can then themselves be partitioned into several areas such that the requests for calls connecting two terminals in the same zone (area) are more likely than those connecting terminals in different zones (areas). A multi-zone-balanced network can then be constructed by replacing the distribution network M_s in Section 4 by the right half of a smaller zone-balanced network.

Chung and Hwang[1] first studied the blocking probability of zone-balanced networks. Previously, several special cases of designing zone-balanced networks have been investigated[1,8]. In this paper we construct a large class of zone-balanced network whose linking pattern can be explicitly specified in a surprisingly simple manner.

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