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Title		Document Number
	The Relationship Between Switch Capacity and Network Capacity in Packet-Switched Networks	
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Authors	Name	Organization Number	Location Code/Room	Telephone Number
	F. R. K. Chung	21214	MH 2C-121	582-6509
	R. R. Goldberg	23152	FJ 3A-124	577-5720

A common question in the planning of packet-switched networks concerns the constraints on total network size imposed by the capacity of the individual nodes.

This memorandum pursues an approach to answering that question relative to the packet-switching networks envisioned for providing public service in the BOCs.

1. BOC Requirements

Reference [1] provides network capacity requirements of approximately 25,000 busy-hour calls. Using an estimate of 0.33 packets per second per call and call holding times in a range from 20 minutes to several hours gives from 2500 to 8000 busy-hour packets per second as a network throughput requirement.

A number of BOC RFPs for packet-switching networks have specified network designs with a maximum end-to-end (one-way) packet delay of 300 ms. average.

In this memorandum we raise the issue of whether it is always possible to build networks of sufficient capacity, while meeting the worst-path delay criterion, from switches with given throughput/delay characteristics.

The approach draws together results from two areas: packet-network design and combinatorial graph theory.

2. Packet Network Design

A Bell Laboratories technical memorandum written in 1982[2] addresses the relationship between packet switching node capacity and packet switching network capacity under certain simplifying assumptions. The conclusion is that if c represents the throughput capacity of a node then the throughput capacity of a fully connected network of s such nodes carrying uniform traffic (the assumptions will be discussed further later) is not cs but rather $\frac{cs}{2-\frac{1}{s}}$.

One way to look at this result is that, for a network made up of more than a few nodes of capacity c, each node contributes only about 0.5 to 0.6 times c to the capacity of the network.

2.1 The Simplifying Assumptions

The two obvious objections to be raised at this point concern the uniform traffic assumption and the fully-connected-network assumption. We will argue that these simplifications influence the result in opposite directions, admitting that the magnitudes are not yet understood.

The meaning of the uniform traffic assumption is that traffic entering the network at each packet switch is equally likely to exit the network at any packet switch (including the one where it entered.) The effect of this assumption is probably to ignore the existence of communities of interest within individual switches. Communities of interest reduce multiple switching, and so the 0.5 to 0.6 factor for derating switch capacity would be unrealistically low.

The fully-connected network assumption is used to assert that a packet is switched at most twice, leading to 0.5 as the worst-case derating factor. Relaxing this assumption allows for triple- or more switching of a single packet, in which case the 0.5 to 0.6 factor might be unrealistically high.

3. Network Topology Constraints

The throughput capacity of a switch constrains the number of other switches it can connect to. The relationship depends also on trunk occupancy, and is demonstrated in the following example: if a switch can support a throughput of 100 packets per second (pps) and trunk occupancy is such that each trunk imposes a load of 33 pps on the node then the node can be connected to just 3 other nodes. If the trunk occupancy were lower, so that each trunk imposed a load of 25 pps on the node, then that node could be connected to 4 others. If the network is viewed as a graph, this property is known as a degree constraint for the graph.

Because of the BOC cross-network delay requirement, the delay property of a switch also constrains the network graph. The sum of cross-switch delays and queueing and transmission delays on each network link must be less than the network delay requirement for the shortest path between each node-pair of the graph. A constraint on the maximal shortest path length between any two nodes in a graph is called a diameter constraint for the graph. As is the case for the throughput-degree relationship, we note that the delay-diameter relationship is dependent on trunk occupancy, which determines the trunk queueing factor.

A graph which has both degree and diameter constraints is finite in size. It is therefore conceivable that such a constrained network could not grow large enough to handle the required throughput at the desired delay level. The question now is to relate switch throughput and delay to maximum network size, using the concepts of degree and diameter constraints.

3.1 Formulating the Graph Constraints

To get the degree constraint we first choose a trunk utilization u and calculate the load imposed on a switch by a trunk. We assume 128-octet average data packet length, and 15% line and protocol overhead, so that a 56Kbps channel has theoretical ($u=1$) one-way capacity of about 48 data packets per second (dpps).

Supposing for a moment that the 56 kbps trunk were carrying 48 dpps in each direction, the throughput capacity of the switch that this load would use up depends on the mix of originating/terminating traffic with tandem traffic. Tandem traffic gets switched through to another trunk, so if a packet were counted as being switched once on this trunk it would be counted again on the other trunk. We must count each tandem packet as half a packet to avoid this double counting. A well-studied number for the mix is not at hand, so for now we will arbitrarily choose one third each of originating, terminating, and tandem packets. Now our trunk at 100% utilization uses up 80 dpps of switch throughput capacity.

This gives us the formula :

$$\textit{degree constraint} = \left\lfloor \frac{c}{80u} \right\rfloor .$$

The diameter constraint arises from delay considerations, both through the switch and through the transmission links. Cross-switch delay (denoted here by d) varies from switch to switch and with varying loads. In general, cross-switch delays for current switching products loaded to near capacity are in the range from 20 to 80 ms.

Transmission link delays depend on link speed and occupancy. Again assuming 128-octet packets and 56 Kbps. links, the service time for a packet is about 20 ms. The queuing delay on the link is approximately $\frac{u}{1-u}$ service times.

To obtain the diameter constraint, using the previously mentioned 300 ms. cross-network delay objective, we first realize that for a given diameter, the associated path will include a number of links equal to the diameter and a number of nodes which is one greater. We account for the extra node by subtracting d from 300, then solve for the diameter by dividing by the delays for each link and node. The formula for diameter constraint is:

$$\textit{diameter constraint} = \left\lfloor \frac{300-d}{\frac{20}{1-u}+d} \right\rfloor .$$

Applying this formula to a reasonable range of u and d values we find a worst-case constraint (80 ms. cross-switch, $u = 0.7$) of less than 2 (i.e. two links and three switches in the path) and a best-case constraint (20 ms. cross-switch, zero trunk queuing) of 7 (7 links and eight switches).

3.2 Network Bounds

The problem of determining the maximum number, denoted by $n(k,D)$, of vertices in a graph having diameter $\le D$ and degree $\le k$ is one of the oldest problems in extremal graph theory. It has received quite a lot of attention in the literature. ([3], [4], [5], [6]) However this neatly formulated problem turns out to be quite difficult. Relatively few exact values of $n(k,D)$ are known so far. For most values of k and D, there are some general upper and lower bound techniques, which we will soon describe. In general these bounds have substantial gaps which notably need further research.

The maximum number $n(k,D)$ of vertices in a graph with diameter D and maximum degree k can be bounded above by

$$1 + k + \ldots + k(k-1)^{D-1} = n_0(k,D)$$

since there are at most $k(k-1)^{i-1}$ vertices at a distance $i \ge 1$ from a vertex.

The upper bound $n_0(k,D)$, called the Moore bound, is provably unreachable [7] for almost all nontrivial values of k and D. The only graphs, called Moore graphs, which achieve the

Moore bounds are[7]:

i. D = 1, (k+1)-cliques

ii. k = 2, (2D+1)-cycles

iii. D = 2 and k = 3, the Peterson graph

iv. D = 2 and k = 7, the Hoffman-Singleton graph

v. (possibly) D = 2 and k = 57.

The current known bounds for $n(k,D)$ for general k and D are indeed very poor. For the upper bound, the only result beyond $n(k,D) < n_0(k,D)$ (except for i-v above) was obtained by P. Erdos, S. Fajtlowicz and A. J. Hoffman[8] who proved

$$n(2k,2) \le n_0(2k,2) - 2 \text{ for } k > 1.$$

It is not known whether it is true that for every integer m there exist k and D such that

$$n(k,D) \le n_0(k,D) - m.$$

There are two different approaches for establishing the lower bounds for $n(k,D)$: explicitly construct such a good graph or prove by probabilistic methods the existence of a good graph. For practical concerns the first approach is more desirable although the second often gives better bounds (so far).

For the purposes of this memorandum, which is concerned with the question of constructing actual networks, we will draw upon the literature for known constructions of graphs with given degree and diameter.

Figures 1, 2, and 3 divide the space of switch throughput versus cross-switch delay into regions of corresponding degree and diameter constraints. Each figure assumes a different value of average trunk occupancy u. All the figures use the formulas given earlier for deriving the constraints. In each degree-diameter cell of each figure is printed the size of the largest known graph having that degree and diameter, as drawn from[9].

4. Implications and Conclusions

The figures can be used to estimate the network capacity attainable with a switch of known throughput and delay characteristics. (The reader is reminded to derate capacity by approximately 0.5 to account for multiple switching.) A range of network capacities is obtained by consulting all three figures. In this way it becomes clear that the trunk occupancy u is really a very critical parameter.

One may also note that these results are rather sensitive to changes in the diameter constraint. They were computed based on a 300 ms. worst-case cross-network delay, but it is not easy to argue that relaxation of that constraint is an unacceptable way to increase network capacity. Therefore some flexibility of thought is demanded when using these figures.

Even with these reservations, the calculations presented here do support several conclusions:

- extremely small packet switches, say those having under 50 dpps. of capacity at 20 ms. delay, may be unable to be configured into networks handling several thousand dpps. without compromising delay objectives.

- Diseconomies in the form of low trunk utilization may be a hidden cost of small packet switches.
- Approaching the known lower bounds given here should be treated with caution. The configuration of even those networks known to exist may be difficult or expensive, since some of these bounds represent unique constructions which may not be desirable from a BOC network point of view. Also there is no guarantee that constructions exist for networks slightly smaller than those described in the literature. There may be a large jump from the size of the largest known graph of given degree and diameter to the next largest such graph.

F. R. K. Chung

R. R. Goldberg

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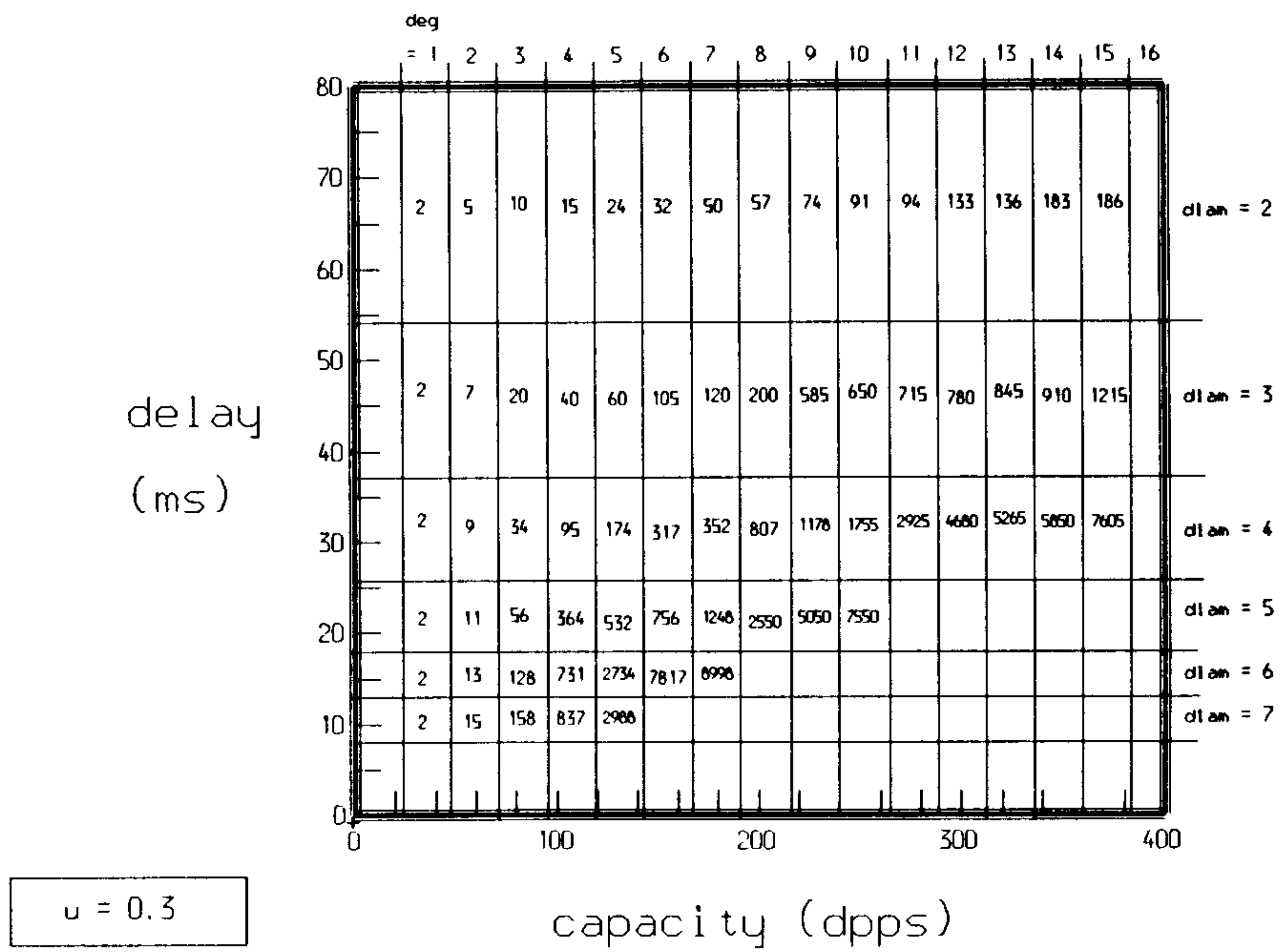

deg
= 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
delay
(ms)
80 70 60 50 40 30 20 10 0
2 5 10 15 24 32 50 57 74 91 94 133 136 183 186
diam = 2
2 7 20 40 60 105 120 200 585 650 715 780 845 910 1215
diam = 3
2 9 34 95 174 317 352 807 1178 1755 2925 4680 5265 5850 7605
diam = 4
2 11 56 364 532 756 1248 2550 5050 7550
diam = 5
2 13 128 731 2734 7817 8998
diam = 6
2 15 158 837 2988
diam = 7
0 100 200 300 400
capacity (dpps)
u = 0.3

Figure 1

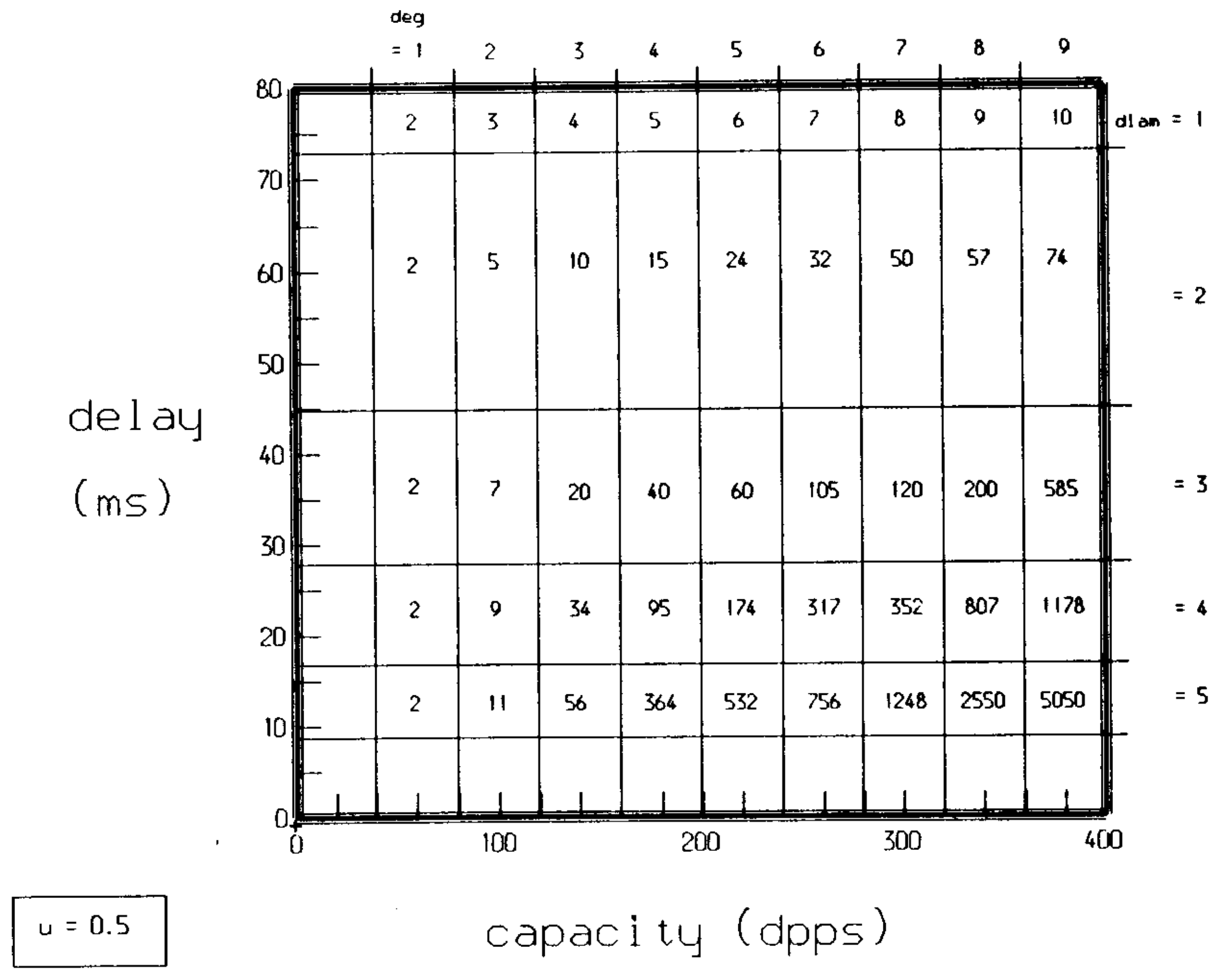
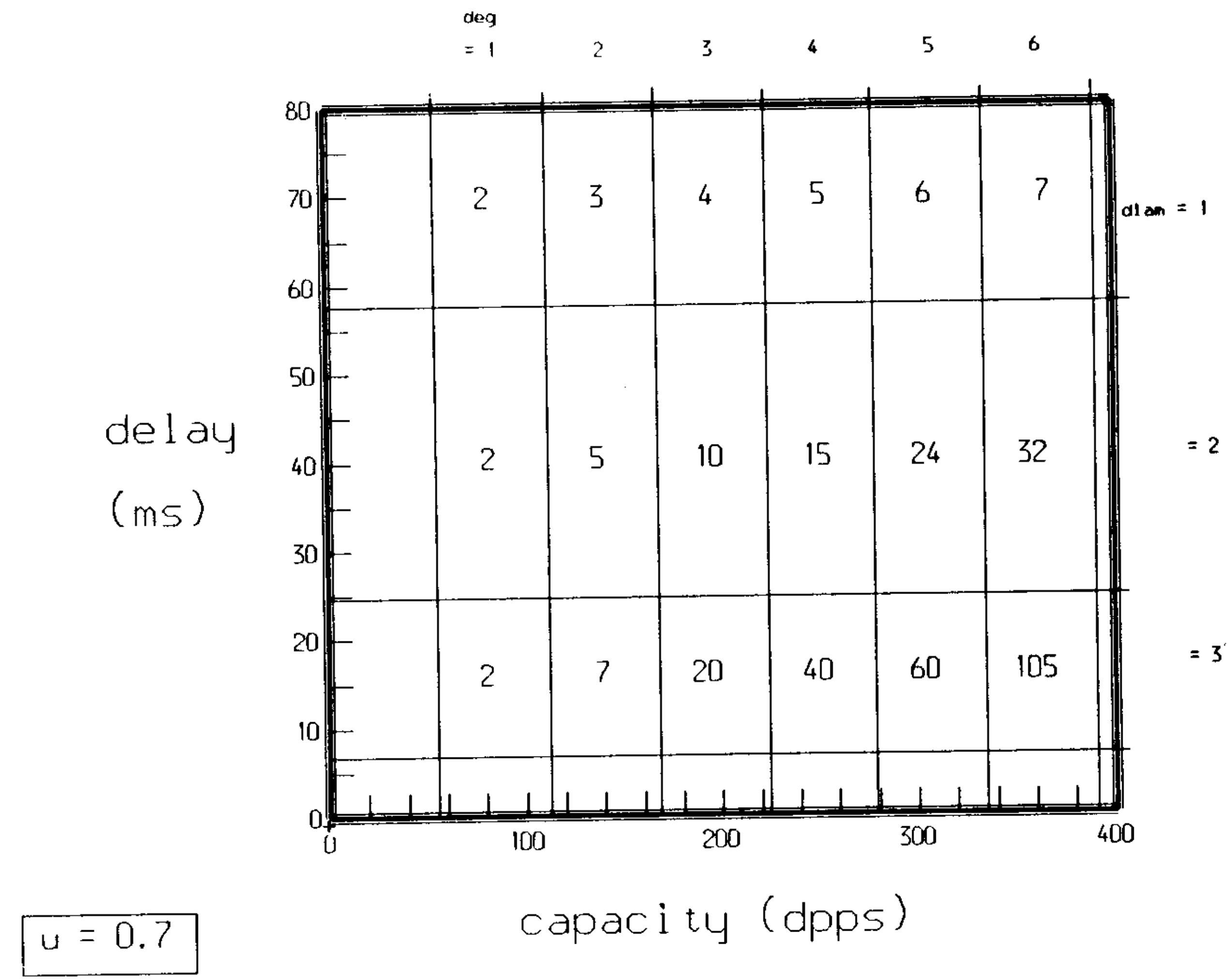

deg
= 1
2
3
4
5
6
7
8
9
diam = 1
= 2
= 3
= 4
= 5
2 3 4 5 6 7 8 9 10
2 5 10 15 24 32 50 57 74
2 7 20 40 60 105 120 200 585
2 9 34 95 174 317 352 807 1178
2 11 56 364 532 756 1248 2550 5050
80
70
60
50
40
30
20
10
0
0
100
200
300
400
delay
(ms)
capacity (dpps)
u = 0.5

Figure 2



deg
= 1
2
3
4
5
6
dlam = 1
= 2
= 3
2
3
4
5
6
7
2
5
10
15
24
32
2
7
20
40
60
105
80
70
60
50
40
30
20
10
0
0
100
200
300
400
delay
(ms)
capacity (dpps)
u = 0.7

Figure 3