

from that of Gaussian. It may be noted that this characteristic of pseudo-error detectors was predicted by Gooding<sup>(3)</sup> and may now be regarded as being confirmed.

Scrutiny of Figures 8, 9 and 10 indicates that the achieved stability of the multiplication factor is sufficient to estimate BER from the pseudo-error rate with an accuracy of about a factor of 10 in BER. In addition, the estimate is performed on in service data in a very modest period of time. For example, the time required to accumulate 100 pseudo errors (more than adequate to obtain a high degree of confidence in the result) of a 1.544 Mbit/s transmission system operating at  $10^{-8}$  BER when the pseudo error monitor has a  $10^5$  multiplication factor at that BER is:

$$t_o = \frac{N_e}{B \cdot \text{BER} \cdot G} = 100 / (1.544 \times 10^6 \times 10^{-8} \times 10^5) \\ = 60 \text{ ms}$$

This expression was obtained from equations (1) and (7) with  $N_e = 100$ ,  $B = 1.544 \cdot 10^6$  bits/s,  $\text{BER} = 10^{-8}$  and pseudo-error detection gain  $G = 10^5$ . The results of the testing on burst type interference show that a very high probability of burst detection can be achieved if the length of the phenomena causing the burst in the 1.544 Mbit/s data is greater than  $40 \mu\text{s}$ . This immediately suggests that in a system with two transmission channels available and a switch to select the

superior that a combination of pseudo-error detectors and data buffers in each channel can be used to avoid errors. Note that some degree of error avoidance would be achieved even if the perturbation is less than  $40 \mu\text{s}$  in duration.

In conclusion, it may be stated that the experimental testing carried out has confirmed the utility of the pseudo-error technique predicted in earlier theoretical papers.

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## Correspondence

### Optimal Multistage Switching Networks

F. R. K. CHUNG

**Abstract**—We consider the problem of determining the multistage network with the fewest crosspoints for given sizes of input and output terminal sets, traffic load, number of stages and blocking probability. In this paper, we present a solution for this problem when the sizes of the input and output terminal sets are greater than a certain value.

#### I. INTRODUCTION

We consider a  $t$ -stage switching network with the set of input terminals  $I$  and the set of output terminals  $\Omega$ . We assume all switches in the same stage have the same number of inlet lines and the same number of outlet lines. To be specific, we assume that stage  $i$  consists of  $s_i$  switches of size  $n_i \times m_i$ ,  $1 \leq i \leq t$ . For an input terminal  $u$  and an output terminal  $v$ , we define the *channel graph* of  $u$  and  $v$ , denoted by  $G(u, v)$  to be the union of all paths connecting  $u$  and  $v$ . (A channel graph

is also called a linear graph [10].) A switching network is said to be *balanced* if all channel graphs  $G(u, v)$ ,  $u \in I$ ,  $v \in \Omega$ , are isomorphic. A channel graph is said to be *regular* if either it is a series combination of smaller regular channel graphs or it is a parallel combination of identical copies of a smaller regular channel graph. A balanced network is said to be a *regular network* if its unique channel graph (up to isomorphism) is regular.

We will assume that the traffic offered to each input terminal is *pure chance traffic* so that the traffic load of each input terminal is independent of the number of busy input terminals (see [2], [11] for references). We will also use Lee's blocking probability model [10] together with his independence assumption that the probabilities of being busy for links in successive stages are independent.

In a regular network with pure chance traffic, the (Lee) blocking probability for the unique channel graph can be taken as the blocking probability of the switching network. In this paper we shall restrict ourselves to regular networks. Almost all networks in current use are either regular or can be decomposed into a small number of regular networks.

We consider the following problem:

For given blocking probability and traffic loads, number of stages, and sizes of input and output terminals, what is the

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structure of the optimal switching network (i.e., the switching network with the least number of crosspoints)?

We will give solutions to this problem for certain ranges of the given parameters. The remaining part of the problem will be taken care of in a subsequent paper. It turns out that it is sometimes impossible to construct a network with its parameters having certain values because of the integer constraints. In that case, the best we can do is to present a few nearly optimal switching networks with given sizes of input and output terminals, traffic load and number of stages and with blocking probability close to the given blocking probability.

Two regular switching networks are not comparable unless they have the same sizes of input and output terminals and the same traffic load. In this case, a switching network  $A$  is said to be *better* than another switching network  $B$  if either (i)  $A$  and  $B$  have the same number of crosspoints but the blocking probability in  $A$  is less than the blocking probability in  $B$ ; or (ii)  $A$  and  $B$  have the same blocking probability but the number of crosspoints in  $A$  is less than the number of crosspoints in  $B$ .

## II. CROSSPOINTS PER ERLANG

We consider a  $t$ -stage switching network as shown in Fig. 1. Let us suppose each input terminal carries  $p_1$  Erlangs of traffic. Let  $p_i$  denote the traffic load for a link between stage  $i-1$  and stage  $i$ ,  $i = 2, 3, \dots, t$ , and  $p_{t+1}$  denote the traffic carried by an output terminal. Furthermore, we write  $|I| = N$ ,  $|\Omega| = M$ . Since the traffic carried by inlet lines of a rectangular switch must be equal to the traffic carried by outlet lines of the same switch, we have the following equations:

$$p_i n_i = p_{i+1} m_i, \quad \text{for } 1 \leq i \leq t \quad (1)$$

$$N p_1 = M p_{t+1}. \quad (2)$$

From (1) and (2) we obtain

$$\frac{n_1 n_2 \cdots n_t}{N} = \frac{m_1 m_2 \cdots m_t}{M}. \quad (3)$$

The total number of crosspoints in the switching network in Figure 1 is given by

$$n_1 m_1 s_1 + n_2 m_2 s_2 + \cdots + n_t m_t s_t$$

where  $s_i$  is the number of switches in stage  $i$ .

The total traffic carried by this network is  $N p_1$  Erlangs. Thus the crosspoints per Erlang (denoted by C.P.E.) is equal to the quotient of the total number of crosspoints divided by  $N p_1$ , i.e.,

$$\text{C.P.E.} = \frac{m_1}{p_1} + \frac{m_2}{p_2} + \cdots + \frac{m_t}{p_t}. \quad (4)$$

Lotze [11] first introduced the C.P.E. method for dealing with switching networks. He used a rough measure, called *transparency*, for measuring the blocking properties of the switching networks. Transparency is defined to be the average number of paths without containing blocked lines from one fixed input terminal to any output terminal. In general, transparency is not a very accurate measure for blocking properties and the relation between transparency and the actual blocking probabilities has not been clearly established. In some cases, other factors have to be considered simultaneously to avoid unexpected blocking. In [11], Lotze investigated optimal

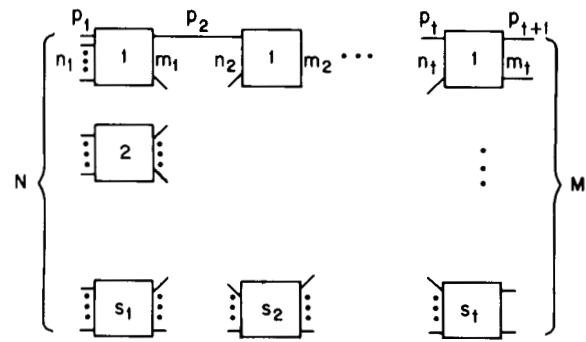


Figure 1. A  $t$ -stage switching network.

switching networks with respect to transparency. Akimarn and Iida [1] studied optimal network designs for 4, 6, or 8 stage networks with respect to Lee blocking probability in a special class of networks. In [12], Lotze and others prepared 4, 5 and 6 stage minimum crosspoint designs where the blocking probabilities are approximated by simulation. Ikeno [8] and Pippenger [12] have studied the asymptotic behavior of the number of crosspoints of optimal networks with square switches as the amount of traffic becomes very large and as the Lee blocking probability becomes very small. However, their optimal networks usually require a large number of stages which might heavily increase the cost of a switching system's common control. Feiner and Kappel [4] introduced the concept of control cost together with crosspoint minimization in deriving optimal networks, and they approximated blocking probability by an access factor related to transparency. Takagi [14, 15] considers switching networks of a specified type and finds optimal ones.

In this paper we minimize C.P.E. with respect to Lee blocking probability and we describe the structure of the optimal switching networks by finding  $m_i$ ,  $n_i$  and  $p_i$ .

## III. THREE-STAGE SWITCHING NETWORKS

Fontenot [5] has derived a method for finding the optimal 3-stage switching networks. In this section we present a simpler scheme for constructing optimal regular 3-stage networks subject to fixed sizes of inlet and outlet terminals, given blocking probability and traffic load.

A channel graph of a regular 3-stage network is illustrated in Fig. 2. Since the network is regular, the parameters  $\gamma_1, \gamma_2, k$  must satisfy the following equations: ( $\gamma_i$  is the number of links between a switch in stage  $i$  to a switch in stage  $i+1$ )

$$\gamma_1 \gamma_2 k = \frac{m_1 m_2 m_3}{M} = \frac{n_1 n_2 n_3}{N}. \quad (5)$$

The Lee blocking probability  $P$  of this network is equal to

$$P = (1 - (1 - p_2^{\gamma_1})(1 - p_3^{\gamma_2}))^k.$$

Thus

$$\log P = \frac{m_1 m_2 m_3}{\gamma_1 \gamma_2 M} \log (1 - (1 - p_2^{\gamma_1})(1 - p_3^{\gamma_2})). \quad (6)$$

Define the function  $f$  by

$$f(m_2, m_3, p_2, p_3, \gamma_1, \gamma_2) = \frac{m_1}{p_1} + \frac{m_2}{p_2} + \frac{m_3}{p_3}.$$

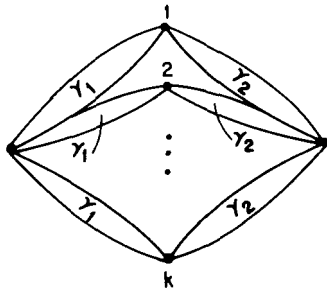


Figure 2. A channel graph of a regular 3-stage network.

Thus,

$$f(m_2, m_3, p_2, p_3, \gamma_1, \gamma_2) = \frac{\gamma_1 \gamma_2 M \log P}{p_1 m_2 m_3 \log(1 - (1 - p_2^{\gamma_1})(1 - p_3^{\gamma_2}))} + \frac{m_2}{p_2} + \frac{m_3}{p_3} \tag{7}$$

We consider  $\partial f / \partial \gamma_1$ . A straightforward calculation shows that  $\partial f / \partial \gamma_1 > 0$  for  $\gamma_1 \geq 0$  and for fixed  $p_2, p_3$  where  $0 < p_2, p_3 < 1$ . Thus we may assume  $\gamma_1 = 1$  as long as the set of parameters for optimal networks we obtain is realizable. Similarly, we may also assume  $\gamma_2 = 1$ . We then set

$$f(m_2, m_3, p_2, p_3) = f(m_2, m_3, p_2, p_3, 1, 1) = \frac{M \log P}{p_1 m_2 m_3 \log(1 - (1 - p_2)(1 - p_3))} + \frac{m_2}{p_2} + \frac{m_3}{p_3}$$

$f$  is minimized when

$$\frac{\partial f}{\partial m_2} = \frac{\partial f}{\partial m_3} = \frac{\partial f}{\partial p_2} = \frac{\partial f}{\partial p_3} = 0.$$

By straightforward calculation, we have  $m_2 = m_3, p_2 = p_3 = p$  and

$$1 - p + (2 - p) \log(2p - p^2) = 0. \tag{8}$$

The value of  $p, p \neq 1$ , which satisfies (8) is  $p = .4547 \dots$ . The sizes of switches in the optimal 3-stage network will be as follows:

$$m_2 = m_3 = n_2 = n_1 = 1.0882(M \log(1/P)/p_1)^{1/3} \tag{9}$$

$$m_1 = \frac{M}{N} n_3 = 2.3931(M \log(1/P))^{1/3} p_1^{2/3} \tag{10}$$

$$\text{C.P.E.} = 7.1793(M \log(1/P)/p_1)^{1/3} \tag{11}$$

Since the switching networks we considered have the property that any input terminal has access to any output terminal, we then have  $m_1 m_2 m_3 / M = k \geq 1, 2.8339 M(-\log P) \geq M$ , i.e.,  $P \leq .7027$ .

From a practical point of view, .7027 is rather high for a blocking probability of a switching network. (Practical switching

networks usually have blocking probabilities less than .1; see [5], [11].) Thus we may restrict ourselves to switching networks with blocking probability  $P \leq .7027$ . Furthermore,  $m_1$  and  $n_3$  have to satisfy the following:

$$\min(m_1, n_3) \geq k = \frac{m_1 m_2 m_3}{M}$$

i.e.,

$$\min(N, M) \geq 1.6604 \left( \frac{\log P}{p_1} \right)^2.$$

We may assume  $N \geq M$  without loss of generality. It suffices to have

$$\frac{\sqrt{M} p_1}{\log(1/P)} \geq 1.2884. \tag{12}$$

We note that in (9)-(12) the parameters  $m_i, n_i$  are not necessarily integers. In fact, a set of parameters  $\{m_1, m_2, m_3, n_1, n_2, n_3, M, N\}$  is realizable (i.e., there exists a regular network with this set of parameters) if and only if the following hold:

(a)  $N, M, n_i, m_i, 1 \leq i \leq 3$ , are integers; (13)

(b)  $\frac{n_1 n_2 n_3}{N} = \frac{m_1 m_2 m_3}{M} \leq \min(m_1, n_3)$ ; (14)

(c)  $\frac{N}{n_1}, \frac{N m_1 m_2}{n_1 n_2 n_3}$  and  $\frac{N m_1 m_2}{n_1 n_2 n_3}$  are integers. (15)

A set of parameters satisfying (13)-(15) is called a realizable set of parameters. In [6], methods are given for constructing a regular 3-stage switching network having any given realizable set of parameters. Thus, in order to find the optimal switching network, we have to choose a realizable set of parameters as close to (9)-(12) as possible. Sometimes, we can arrive at more than one alternative. The following example illustrates the design scheme of a nearly optimal network.

*An example*

Suppose we want to design an optimal network with  $N = 128, M = 80, P = .05, p_1 = .6$ . First, we know that

$$\frac{\sqrt{M} p_1}{\log(1/P)} = 1.791 > 1.288.$$

Thus, by (8)-(11) we have  $m_2 = m_3 = n_1 = n_2 = 8.012, m_1 = 10.574, n_3 = 16.918, \text{C.P.E.} = 52.872$ . Then we choose a realizable set of parameters with values close to the above values as follows:  $m_2 = m_3 = n_1 = n_2 = 8, m_1 = 10, n_3 = 16$ .

A switching network can then be constructed according to this set of parameters as shown in Fig. 3. We note that links between the second and third stage are connected cyclically. This network has  $\text{C.P.E.} = 50$  and with blocking probability  $P = .0803$ . If this blocking probability is too high to be acceptable, we could use the following alternative.  $n_1 = n_2 = m_3 = 8, m_2 = 9, m_1 = 10, n_3 = 18$ .

It can be easily checked that this is a realizable set of parameters. The switching network with this set of parameters is illustrated in Fig. 4. This switching network has blocking

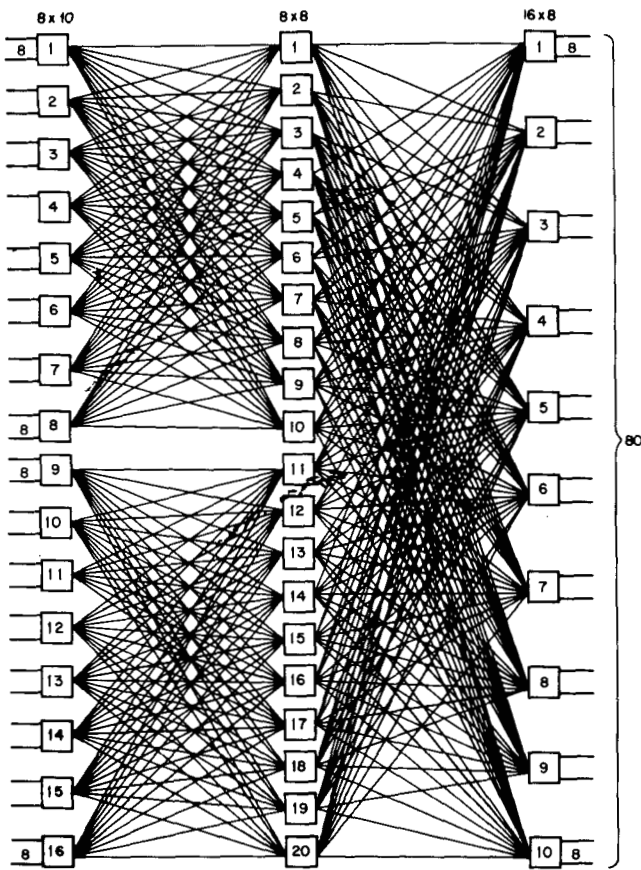


Figure 3. An example.

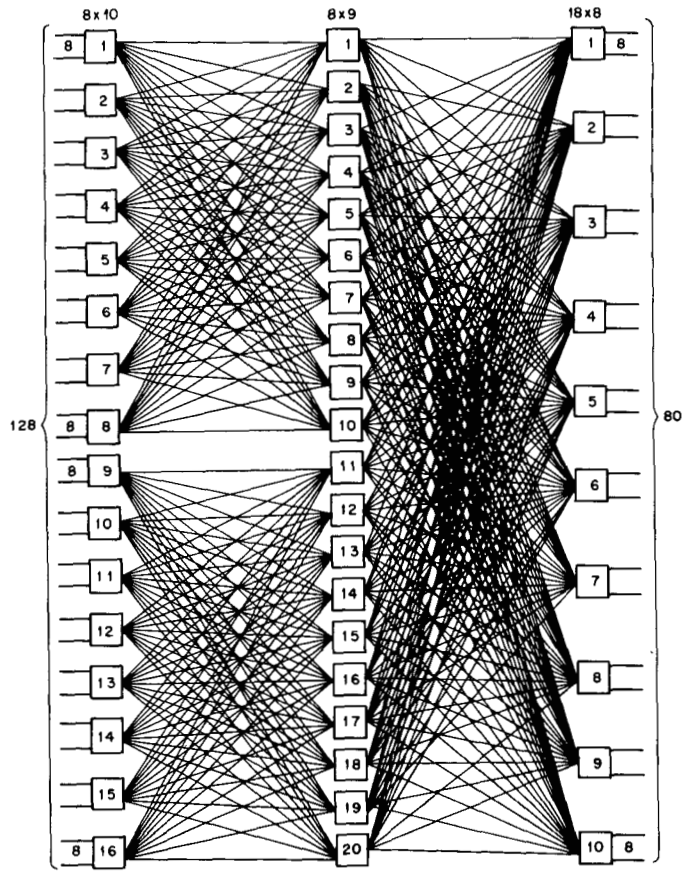


Figure 4. An example.

probability .0413 and with C.P.E. = 54.166. The choice between the networks in Fig. 3 and 4 depends on practical considerations.

IV. MULTI-STAGE SWITCHING NETWORKS

Unlike 3-stage channel graphs, there is a large set of non-isomorphic  $t$ -stage channel graphs for  $t > 3$ . Let  $Q_{t, k}$  be the set of  $t$ -stage channel graphs which are the unions of  $k$  distinct paths. In  $Q_{t, k}$ , there is a unique channel graph called a  $k$ -spread channel as shown in Fig. 5. Suppose  $A$  is a regular switching network with a channel graph in  $Q_{t, k}$ . If the channel graph of  $A$  is not the  $k$ -spread channel graph, the linking pattern of  $A$  can sometimes be changed (when  $m_1 \geq k, n_t \geq k$ ) so that the new switching network has the same number of crosspoints but with smaller blocking probability (see [3], [7]).

In order to simplify the analysis of optimal  $t$ -stage networks, we only consider  $k$ -spread channel graphs and with given parameters (size of input and output terminals, traffic loads, blocking probabilities) satisfying certain constraints which we will describe specifically later.

We consider a  $t$ -stage regular network  $A$  with a  $k$ -spread channel graph. Thus the blocking probability  $P$  of this network is given by

$$P = (1 - (1 - p_2(1 - p_3) \dots (1 - p_t)))^k$$

and

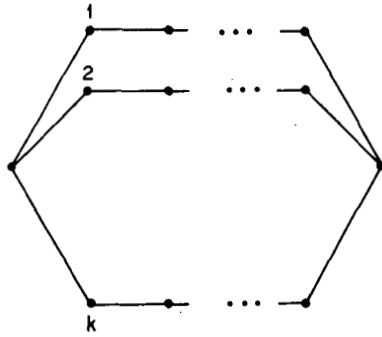
$$k = \frac{m_1 m_2 \dots m_t}{M} = \frac{n_1 n_2 \dots n_t}{N}$$

Now we consider the C.P.E. of this network. We have

$$\begin{aligned} f(m_2, m_3, \dots, m_t, p_2, p_3, \dots, p_t) &= \frac{m_1}{p_1} + \frac{m_2}{p_2} + \dots + \frac{m_t}{p_t} \\ &= \frac{M \log P}{p_1 m_2 m_3 \dots m_t \log (1 - (1 - p_2)(1 - p_3) \dots (1 - p_t))} \\ &\quad + \frac{m_2}{p_2} + \frac{m_3}{p_3} + \dots + \frac{m_t}{p_t} \end{aligned}$$

The minimum value of  $f$  is achieved when  $\partial f / \partial m_i = 0, \partial f / \partial p_i = 0, 2 \leq i \leq t$ . Therefore, by setting  $\partial f / \partial m_i = 0, \partial f / \partial p_i = 0, 2 \leq i \leq t$ , we have the following:

$$\begin{aligned} \frac{m_2}{p_2} = \dots = \frac{m_t}{p_t} &= \frac{M \log P}{p_1 m_2 \dots m_t \log (1 - (1 - p_2)(1 - p_3) \dots (1 - p_t))} \\ &= \frac{M p_2 \log P (1 - p_3) \dots (1 - p_t)}{p_1 m_2 \dots m_t (\log (1 - (1 - p_2)(1 - p_3) \dots (1 - p_t)))^2 (1 - (1 - p_2)(1 - p_3) \dots (1 - p_t))} \end{aligned}$$

Figure 5. A  $k$ -spread channel graph.

We then have

$$m_2 = m_3 = \dots = m_t = m,$$

$$p_2 = p_3 = \dots = p_t = p,$$

and

$$p(1-p)^{t-2} + (1 - (1-p)^{t-1}) \log(1 - (1-p)^{t-1}) = 0. \quad (16)$$

We note that  $p = \tilde{p}_t$  approaches .5 when  $t$  approaches infinity. In Table 1, values of  $\tilde{p}_t$  are listed for  $t < 10$ .

The sizes of the switches in the optimal network will be as follows:

$$\begin{aligned} m_2 = m_3 = \dots = m_t = n_1 = \dots = n_{t-1} \\ = \left( \frac{-M \log P}{p_1} \right)^{1/t} \cdot \left( \frac{p}{-\log(1 - (1-p)^{t-1})} \right)^{1/t} \\ = c_t (M \log(1/P)/p_1)^{1/t}, \end{aligned} \quad (17)$$

$$\begin{aligned} m_1 = \frac{M}{N} n_t = \left( \frac{-M \log P}{p_1} \right)^{1/t} \cdot \frac{p_1}{p} \\ \cdot \left( \frac{p}{-\log(1 - (1-p)^{t-1})} \right)^{1/t} \\ = \frac{p_1}{p} c_t (M \log(1/P)/p_1)^{1/t}, \end{aligned} \quad (18)$$

$$\begin{aligned} \text{C.P.E.} = \frac{t}{p} \left( \frac{-M \log P}{p_1} \right)^{1/t} \cdot \left( \frac{p}{-\log(1 - (1-p)^{t-1})} \right)^{1/t} \\ = \frac{t}{p} c_t (M \log(1/P)/p_1)^{1/t}. \end{aligned} \quad (19)$$

The values of  $c_t$  are listed in Table 1. If we approximate  $p$  by .5, for  $t \geq 10$ , then we have the following:

$$m_2 = m_3 = \dots = m_t = n_1 = \dots = n_t = (M \log(1/P)/p_1)^{1/t} \cdot 2(.25 - (.5)^{t+1})^{1/t}$$

$$m_1 = \frac{M}{N} n_t = (M \log(1/P)/p_1)^{1/t} 4p_1 (.25 - (.5)^{t+1})^{1/t}$$

$$\text{C.P.E.} = t(M \log(1/P)/p_1)^{1/t} 4p_1 (.25 - (.5)^{t+1})^{1/t}.$$

TABLE 1  
VALUES OF  $\tilde{p}_t$ ,  $c_t$  AND  $d_t$ 

$t$	$\tilde{p}_t$	$c_t$	$d_t$
3	.45471	1.0882	1.6604
4	.48093	1.33669	$3.2531 \times 10^1$
5	.49128	1.47942	$2.5223 \times 10^3$
6	.49590	1.57010	$7.5547 \times 10^5$
7	.49795	1.63211	$8.6142 \times 10^8$
8	.49921	1.67814	$3.9157 \times 10^{12}$
9	.49922	1.71166	$6.3394 \times 10^{16}$

Since we assume  $P \leq .7027$ , it is easy to check that  $(m_1 \dots m_t)/M \geq 1$  from (17)-(19). Thus any input terminal has access to any output terminal. Moreover,  $m_1, n_t$  have to satisfy the following:

$$\begin{aligned} \min(m_1, n_t) &\geq k = \frac{m_1 m_2 \dots m_t}{M} \\ \min(N, M) &\geq \left( \frac{\log(1/P)}{p_1} \right)^{t-1} \left( \frac{p}{-\log(1 - (1-p)^{t-1})} \right)^{t-1} \\ &= d_t \left( \frac{\log(1/P)}{p_1} \right)^{t-1}. \end{aligned} \quad (20)$$

The values of  $d_t$  are listed in Table 1. For  $t \geq 10$   $d_t$  is approximately  $(1 - (.5)^{t-1}/(.5)^{t-2})^{t-1}$ .

The necessary and sufficient conditions for a set of parameters of multi-stage networks to be realizable are not completely understood. Certain special cases are studied in [5], [6]. However, the optimal network with parameters in (17)-(19) can be constructed by the following method if the integer constraints are satisfied. First, we will suppose that the values of  $m_i, n_i, 1 \leq i \leq n$ , in (17), (18) are integers.

Let us consider parameter set  $\{M, N, m_1', m_2', m_3', n_1', n_2', n_3'\}$  for a 3-stage network, where

$$\begin{aligned} m_1' &= m_1, & n_1' &= n_1, \\ m_2' &= n_2' = m^{t-2}, & \text{where } m &= m_i, & 2 \leq i \leq t \\ m_3' &= m_3, & n_3' &= n_3. \end{aligned}$$

The necessary and sufficient conditions for this set of parameters to be realizable can be easily checked. This set of parameters is realizable if and only if

$$\frac{N}{m'}, \frac{Nm_1}{m^{t-1}'}, \frac{Nm_1}{m^{t-2}n_3'} \quad \text{are all integers.} \quad (21)$$

The switches of the second stage in this network are replaced by copies of a  $(t-2)$ -stage network  $W_{t-2}$  of size  $m^{t-2} \times m^{t-2}$  which is constructed as shown in Fig. 6. The resulting  $t$ -stage network is the optimal regular network we want. If the integer constraints are not satisfied (or  $m$  is not an integer), we will then, as before, construct nearly optimal networks by choosing integers close to the optimal values.

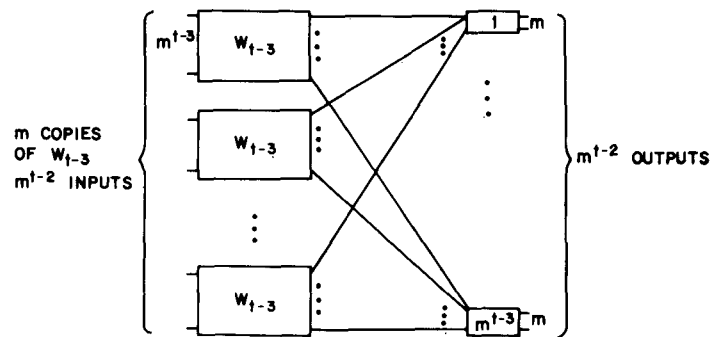


Figure 6.

## V. SUMMARY

For given sizes of input and output terminal sets, given traffic load, number of stages and blocking probability, we have described the structure of the optimal switching network when the sizes of input and output terminal sets are greater than a certain value (see (20) and Table 1). The sizes of the switches ( $m_i, n_i, 1 \leq i \leq t$ ) are given in (17), (18). The linking patterns are described in Sec. III and IV for 3-stage and  $t$ -stages,  $t > 3$ , respectively. Since realizable parameters must all be integers and some basic equalities for networks (see (1), (2), (3), (21)) should be satisfied, we must, if necessary, choose integer values of parameters satisfying (1), (2), (3), (21), and as close to (17), (18) as possible.

As pointed out by J. G. Kappel [9], the optimal switch sizes derived in (17) are in close agreement with some of those currently in use (e.g. those of the 8-stage No. 1 ESS trunk-to-trunk network) which, however, do not actually satisfy the constraints in (20). In general, we would usually have to consider channel graphs of a more general type which of course increases tremendously the complexity of crosspoint minimization. The whole problem of determining optimal networks in full generality is not completely understood at present. Hopefully, this gap in our knowledge can be filled by further research.

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## A Subscriber Carrier System Based Upon Frame Addressing System

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**Abstract**—A new channel assignment scheme called a frame addressing system is proposed in this paper to realize an efficient telephone subscriber carrier system by means of PCM loop having a simplified switching capability at each subscriber. In the frame addressing system, a speech signal time slot is assigned by sending a bit for each subscriber at the beginning of each transmission frame. This results in an improvement of efficiency by a TASI-like effect in addition to efficiency by switching. The principle and an example of a practical circuit are described. Characteristics of the system are analyzed and a design example is described.

## I. INTRODUCTION

In communication systems like a subscriber carrier system or a mobile communication system, a switching circuit for each station must be a simple one to economize cost per subscriber. Several approaches have been proposed for this purpose. The simplest approach is the fixed channel allocation. The approach of the SLM (subscriber loop multiplexer) system in which an ordinary switch is provided for each subscriber has also been proposed and commercially used.<sup>(1)</sup> In this system, switching is performed for each message; thus, it may be called message addressing.

In digital telephone communication systems, speech signals are sampled periodically and transmitted occupying a time slot in a frame. Therefore, by providing an addressing signal in each frame to allocate the time slot for each subscriber in the frame,

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