

Consider the function

$$F(x, y, z) = x^2y + e^x + z$$

and its locus of zeros $Z(F) := \{(x, y, z) \mid x^2y + e^x + z = 0\} \subset \mathbb{R}^3$. Show that $(0, 1, -1) \in Z(F)$ and that there exists a differentiable function g defined in an open neighborhood of $(1, -1)$ such that

$$F(g(y, z), y, z) = 0, \quad g(1, -1) = 0.$$

Furthermore, find the derivative of g at $(1, -1)$.

Write the equation of the tangent vector space to $Z(F)$ at $(0, 1, -1)$.

Write the equation of the tangent affine space to $Z(F)$ at $(0, 1, -1)$.