The norm of a linear transformation L is defined as

$$\|L\| := \mathrm{Sup}_{v \neq 0} \left\{ \frac{|L(v)|}{|v|} \right\}.$$

(a) Prove that $\|L\| = \mathrm{Sup}_{|v|=1} \{|L(v)|\}$.

(b) Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a differentiable function. Suppose that the norm of the derivative of f is bounded. Prove that f is uniformly continuous.