(a) Is the function $g(x) = x^2$ uniformly continuous on $(0, 1)$? Is it uniformly continuous on $\mathbb{R}$? Is the function $f(x) = \frac{1}{x}$ uniformly continuous on $(0, \infty)$? On $(0, 1]$? If $b > a > 0$, is f uniformly continuous on $[a, b]$?

(b) Which of the following sets are compact?

- (i) $K := \{\frac{1}{2^n} \mid n > 0\}$
- (ii) the set of rational numbers in the interval $[0, 1]$
- (iii) a finite union of compact sets
- (iv) an arbitrary union of compact sets
- (v) a finite intersection of compact sets
- (vi) an arbitrary intersection of compact sets
- (vii) the boundary of a bounded set
- (viii) the boundary of an arbitrary set

(c) Extra credit: Let X be a subset of $\mathbb{R}^n$. We say that a subset V of X is open if there exists an open subset U of $\mathbb{R}^n$ such that $V = U \cap X$. We say that a subset Z of X is closed if there exists a closed subset Y of $\mathbb{R}^n$ such that $Z = Y \cap X$. Prove

- (i) a function $f : X \to \mathbb{R}^m$ is continuous if and only if, for every open subset U of $\mathbb{R}^m$, the pre-image $f^{-1}(U)$ is open,
- (ii) a function $f : X \to \mathbb{R}^m$ is continuous if and only if, for every closed subset Y of $\mathbb{R}^m$, the pre-image $f^{-1}(Y)$ is closed.