

MATH 31BH

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February 24 2023: Preparation for the second midterm

The second midterm will cover the material of Sections 1.7, 1.8, 1.9, 2.10, 3.1, 3.2.

Please explain or prove all your assertions and show your work. You are allowed to use all results in the book or done in class, unless the problem specifically says you cannot use a particular result. When using a result, please state it precisely before using it.

- (1) Give the following definitions
  - (a) partial derivatives
  - (b) Jacobian matrix
  - (c) total derivative
  - (d) directional derivative
  - (e)  $C^p$  function, for  $p \in \mathbb{N}$
  - (f) graph of a function  $\mathbb{R}^n \rightarrow \mathbb{R}^m$
  - (g) smooth  $k$ -dimensional manifold in  $\mathbb{R}^n$
  - (h) parametrization of a smooth  $k$ -dimensional manifold
  - (i) tangent vector space to a manifold
  - (j) tangent affine space to a manifold
  - (k)  $C^p$  function on a manifold
  - (l) the derivative of a  $C^1$  map on a manifold
- (2) State the chain rule for derivatives of  $\mathbb{R}^m$ -valued functions of several variables.
- (3) State and prove the mean value theorem for real-valued functions of several variables.
- (4) Prove that a  $C^1$  function is differentiable.
- (5) State the inverse function theorem in several variables (short form or the form we gave in class).
- (6) State the implicit function theorem in several variables (short form or the form we gave in class).
- (7) Consider the system of equations

$$x^2 + y^2 + z^2 - yw = 2, \quad xyw - yz + z^2 + w^2 = 0.$$

Show that  $(0, 1, 1, 0)$  satisfies the system. Show that, in a neighborhood of  $(0, 1, 1, 0)$ , we can solve  $z, w$  in terms of  $x$  and  $y$ :

$$(z, w) = g(x, y)$$

for a function  $g$  with  $g(0, 1) = (1, 0)$ . Find the derivative of  $g$  at  $(0, 1)$ .

- (8) Review all homework exercises. Note that the material covered by Homework 6 is part of the test.