Extra-credit: Suppose f, g are two differentiable functions from the space M_n of $n \times n$ matrices to itself. Prove that the function fg defined by matrix multiplication as $(fg)(A) := f(A)g(A)$ is differentiable and that its differential at $A \in M_n$ is the following linear map

$$D(fg)(A) : M_n \longrightarrow M_n$$

$$V \longmapsto [Df(A)](V)\ g(A) + f(A)\ [Dg(A)](V)$$