MATH 31BH

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March 17 2023: Preparation for the final exam

The final exam is cumulative and covers all the material we learned this quarter. This means Sections 1.5, 1.6, 1.7, 1.8, 1.9, 2.10, 3.1, 3.2, 3.3, 3.4, 3.6, 3.7. We did not cover the fundamental theorem of algebra in Section 1.7, the classification of constrained extrema and the spectral theorem at the end of Section 3.7.

Please explain or prove all your assertions and show your work. You are allowed to use all results in the book or done in class, unless the problem specifically says you cannot use a particular result. When using a result, please state it precisely before using it.

(1) Review the preparations for the first and second midterms.
(2) Review all homework problems.
(3) Give the following definitions
   (a) the Taylor polynomial of degree k of a function of class C^k on an open set $U \subset \mathbb{R}^n$ at a point $a \in U$.
   (b) a function f is a little o of a function h (on some open set $U \subset \mathbb{R}^n$ of $0 \in \mathbb{R}^n$)
   (c) critical point
   (d) critical value
   (e) signature of a quadratic form (i.e., quadratic polynomial)
   (f) signature of a critical point
   (g) negative definite and positive definite quadratic form
   (h) saddle point
   (i) critical point of a function defined on a manifold
   (j) Lagrange multipliers
(4) Prove that if $M \subset \mathbb{R}^n$ is a smooth k-dimensional manifold, then every point p of M has a neighborhood U in $\mathbb{R}^n$ such that there exists a C^1 mapping $F : U \to \mathbb{R}^{n-k}$ with $DF(p)$ surjective and $M \cap U = F^{-1}(0)$ (the set of zeros of F).
(5) Let $U \subset \mathbb{R}^n$ be open and $f : U \to \mathbb{R}$ be a differentiable function. Prove that if $v \in U$ is a local extremum for f, then $Df(v) = 0$.

(6) Let $M \subset \mathbb{R}^n$ be a smooth manifold and $f : M \to \mathbb{R}$ be a C^1 function. Prove that if $v \in M$ is a local extremum for f, then $Df(v) = 0$.

(7) Do the following problems from the book:

3.7.8, 3.7.10, 3.7.11.