1. See pages 212 and 107, respectively.

2. Since columns 1, 2 and 5 are pivot columns, we can compute the dimensions:

$$\dim(\mathrm{Col}\,A) = \dim(\mathrm{Row}\,A) = 3 \quad \text{and} \quad \dim(\mathrm{Nul}\,A) = 2.$$

The first, second and fifth columns of $A$ are a basis for $\mathrm{Col}\,A$.
The nonzero rows of the reduced echelon form are a basis for $\mathrm{Row}\,A$.

A basis for $\mathrm{Nul}\,A$ is $\begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$.

3. (a) There is not enough information because an elementary row operation may multiply a row by a constant. For example $A + I_6$ and the matrix that equals $I_6$ except that $a_{1,1} = -4$ both have $I_6$ for reduced row echelon form. The determinants are 1 and $-4$.

(b) Yes. Reduced echelon form being the identity is equivalent to the existence of the inverse. (Theorem 3.7 or 3.8b)

(c) Yes. We know that the existence of the inverse guarantees unique solutions to $A\vec{x} = \vec{b}$. (Theorem 3.5) There are other arguments, too.

(d) Yes. We are given $\dim(\mathrm{Nul}\,A) = 3$. Hence $\dim(\mathrm{Col}\,A) = 8 - 3 = 5$. Since $\mathrm{Col}\,A$ is the set of possible constant terms and is not all of $\mathbb{R}^6$, inconsistent $\vec{b}$ exist. Instead of the last sentence one could observe that since there are only 5 pivots, the echelon form of $A$ will have $6 - 5 = 1$ rows that are all zero.

4. (a) Let the $i$th component of $\vec{b}_j$ be $b_{i,j}$. The $j$th column of $AB$ is $A\vec{b}_j = b_{1,j}\vec{a}_1 + \ldots + b_{n,j}\vec{a}_n$, a linear combination of the columns of $A$. Hence $A\vec{b}_j$ is in $\mathrm{Col}\,A$.

(b) Since every column of $AB$ is in $\mathrm{Col}\,A$, it follows that $\mathrm{Col}\,(AB)$ is a subspace of $\mathrm{Col}\,A$. Hence $\dim(\mathrm{Col}\,(AB)) \leq \dim(\mathrm{Col}\,A)$. Since $\dim \mathrm{Col}$ is rank, we are done.

(c) We have $\mathrm{rank}\,(AB) = \mathrm{rank}\,((AB)^T) = \mathrm{rank}\,(B^T A^T)$. Now use (b) with $A$ and $B$ replaced by $B^T$ and $A^T$ respectively to obtain $\mathrm{rank}\,(B^T A^T) \leq \mathrm{rank}\,(B^T)$. We also have $\mathrm{rank}\,(B^T) = \mathrm{rank}\,(B)$. Putting all this together, $\mathrm{rank}\,(AB) \leq \mathrm{rank}\,(B)$.