1. (10 pts.) Define the following

    (a) The span of the vectors $\vec{v}_1, \ldots, \vec{v}_k$.

    (b) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$.

2. (30 pts.) Let $A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & -1 \\ 1 & 2 & 1 & 3 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

    (a) Describe all solutions to the equation $A\vec{x} = \vec{0}$.

    (b) Describe all solutions to the equation $A\vec{x} = \vec{b}$.

3. (45 pts.) The reduced row echelon form for the matrix A is $M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

    For each of the following answer either "Yes," "No," or "Not enough information."
    Note that all the questions are about A, *not* about M.
    To receive credit *you must justify your answers*.

    (a) The columns of A are linearly independent.

    (b) The equation $A\vec{x} = \vec{0}$ has more than one solution.

    (c) The equation $A\vec{x} = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ has more than one solution.

4. (15 pts.) Prove the following:

    If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation which is both one-to-one and onto, then $m = n$.

    Hint: Prove that one-to-one implies that $n \le m$.
    Prove that onto implies that $n \ge m$.