- Please put your name and ID number on your blue book.
- The exam is CLOSED BOOK except for one page of notes.
- Calculators are NOT allowed.
- **You must show your work to receive credit.**

1. (16 pts.) A and B are 3×3 matrices, $\det A = 3$ and $\det B = 2$. For each of the following, give its value if you have enough information. If you do not have enough information say "not enough information."

   (a) $\det(A^{-1})$ (b) rank(A) (c) $\det(A + B)$ (d) $\det(A^T B)$

2. (7 pts.) Find the eigenvalues of $A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$.

3. (7 pts.) The linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ -x_1 \end{bmatrix}$. Let $\mathcal{B}$ be the basis $\mathbf{b}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Compute $[T]_{\mathcal{B}}$.

4. (20 pts.) A is a 8×8 matrix with characteristic polynomial $\lambda^2(\lambda - 1)^3(\lambda + 2)^3$. Two of the eigenspaces are each two dimensional—but I don't know which two.

   (a) What values are possible for the dimension of the third eigenspace?

   (b) Is it possible that A is diagonalizable? *To receive credit, you must give a reason for your answer.*

   (c) Is it possible that A is singular? *To receive credit, you must give a reason for your answer.*

   (d) For what values of c is it possible to find a nonzero vector $\mathbf{v} \in \mathbb{R}^8$ such that $A\mathbf{v} = c\mathbf{v}$?

END OF EXAM