1. (a) Yes, because x_2 and x_3 are free variables.
   (b) No, because there is a row of zeros. (When the augmented matrix is put in row echelon form, the last column can be anything, depending on the choice of **b** and so the last equation will become "zero = anything".

2. (a) Undefined: A is 2×3 but A^T is not.
   (b) Undefined: for BC to be defined, the number of rows of C must equal the number of columns of B. With $A = B = C$, this is not true.
   (c) $\begin{bmatrix} 5 & -2 \\ -2 & 2 \end{bmatrix}$.
   (d) Undefined: To have an inverse, a matrix must have the same number of rows and columns.

3. The augmented matrix and reduction to row echelon form:

$$\begin{bmatrix} 1 & -1 & 2 & 2 \\ 2 & 1 & -2 & 4 \\ 1 & -4 & 8 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 3 & -6 & 0 \\ 0 & -3 & 6 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 3 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus x_3 is a free variable. The second row tells us that $x_2 = 2x_3$. The first row tells us that $x_1 = 2 + x_2 - 2x_3 = 2$. Thus we have

$$x_1 = 2 \qquad x_2 = 2x_3 \qquad x_3 \text{ free.}$$

4. (a) all $p \geq 4$ (b) all $p \leq 4$

5. Since the number of columns of A^T equals the number of rows of A, the product is defined. Since A^T has p rows and A has p columns, $A^T A$ is $p \times p$. Recalling that $(BC)^T = C^T B^T$ and $(B^T)^T = B$, we have

$$(A^T A)^T = A^T (A^T)^T = A^T A.$$