1. (6 pts.) The row echelon form of the matrix $A$ is

$$\begin{bmatrix}
\blacksquare & \ast & \ast & \ast & \ast \\
0 & 0 & 0 & \ast & \ast \\
0 & 0 & 0 & 0 & \blacksquare \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

where $\blacksquare$ is any nonzero number and $\ast$ is any number.

(a) Does $Ax = 0$ have nontrivial solutions? You must give a reason to receive credit.

(b) Does $Ax = b$ have at least one solution for every $b \in \mathbb{R}^4$? You must give a reason to receive credit.

2. (12 pts.) Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$. In each case, compute the indicated quantity or explain why it is undefined.

(a) $A + A^T$  
(b) $A^2$  
(c) $AA^T$  
(d) $A^{-1}$.

3. (10 pts.) Write down the augmented matrix for the following linear equations and use it to find all solutions to the equations.

$$\begin{align*}
x_1 - x_2 + 2x_3 &= 2 \\
2x_1 + x_2 - 2x_3 &= 4 \\
x_1 - 4x_2 + 8x_3 &= 2
\end{align*}$$

(To help avoid errors, you can check that your solution works in the equations.)

4. (6 pts.) You need not give reasons in this problem.

(a) For what values of $p$ is it possible to find $v_1, \ldots, v_p \in \mathbb{R}^4$ so that $v_1, \ldots, v_p$ span $\mathbb{R}^4$?

(b) For what values of $p$ is it possible to find $v_1, \ldots, v_p \in \mathbb{R}^4$ so that $v_1, \ldots, v_p$ are linearly independent?

5. (4 pts.) A matrix $B$ is called symmetric if $B^T = B$. Let $A$ be an $n \times p$ matrix. Prove that $A^T A$ is defined and is a symmetric $p \times p$ matrix.

END OF EXAM