- Please put your name and ID number on your blue book.
- The exam is CLOSED BOOK except for one page of notes.
- Calculators are NOT allowed.
- **You must show your work to receive credit.**

1. (6 pts.) The row echelon form of the matrix A is

$$\begin{bmatrix} \blacksquare & * & * & * & * \\ 0 & 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & 0 & \blacksquare \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where $\blacksquare$ is any nonzero number and $*$ is any number.

(a) Does $A\mathbf{x} = \mathbf{0}$ have nontrivial solutions? You must give a reason to receive credit.

(b) Does $A\mathbf{x} = \mathbf{b}$ have at least one solution for every $\mathbf{b} \in \mathbb{R}^4$? You must give a reason to receive credit.

2. (12 pts.) Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$. In each case, compute the indicated quantity or explain why it is undefined.

(a) $A + A^T$ (b) A^2 (c) AA^T (d) A^{-1}.

3. (10 pts.) Write down the augmented matrix for the following linear equations and use it to find all solutions to the equations.

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 2 \\ 2x_1 + x_2 - 2x_3 &= 4 \\ x_1 - 4x_2 + 8x_3 &= 2 \end{aligned}$$

(To help avoid errors, you can check that your solution works in the equations.)

4. (6 pts.) You need not give reasons in this problem.

(a) For what values of p is it possible to find $\mathbf{v}_1, \ldots, \mathbf{v}_p \in \mathbb{R}^4$ so that that $\mathbf{v}_1, \ldots, \mathbf{v}_p$ span $\mathbb{R}^4$?

(b) For what values of p is it possible to find $\mathbf{v}_1, \ldots, \mathbf{v}_p \in \mathbb{R}^4$ so that that $\mathbf{v}_1, \ldots, \mathbf{v}_p$ are linearly independent?

5. (4 pts.) A matrix B is called *symmetric* if $B^T = B$. Let A be an $n \times p$ matrix. Prove that $A^T A$ is defined and is a symmetric $p \times p$ matrix.

END OF EXAM