1. The matrix is $[\mathbf{v}_1\ \mathbf{v}_2\ \mathbf{v}_3]$.

2. (a) Set $L(\mathbf{x}) = \mathbf{0}$ and solve for $\mathbf{x}$.
   For A, $x_2 = x_1$ and $x_3 = -x_1$. So $\ker(L)$ is spanned by $(1, 1, -1)^T$.
   For B, $x_3 = x_1$ and $x_2 = -x_1$. So $\ker(L)$ is spanned by $(1, -1, 1)^T$.

   (b) There are an infinite number of possibilities. The easiest choice (since it takes little thought and little calculation) is probably $L(\mathbf{i})$, $L(\mathbf{j})$ and $L(\mathbf{k})$ since $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. You can compute the actual values.

   (c) A: $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ B: $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$

   (d) Since the dimension of the range plus the dimension of the kernel equals the dimension of the whole space, we have (ans) $+ 1 = 3$ and so the answer is 2.

3. We are given that $B = S^{-1}AS$. Taking inverses, we have

$$B^{-1} = (S^{-1}AS)^{-1} = S^{-1}A^{-1}(S^{-1})^{-1} = S^{-1}A^{-1}S.$$

4. This is the same problem on both exams, but the names have been changed. Suppose X is a subspace of Y is a subspace of $\mathbb{R}^n$. We must prove $Y^\perp$ is contained in $X^\perp$.

   Suppose that $\mathbf{v} \in Y^\perp$. This means that $\mathbf{v}^T\mathbf{w} = 0$ for every $\mathbf{w} \in Y$. Since X is contained in Y, it follows that $\mathbf{v}^T\mathbf{w} = 0$ for every $\mathbf{w} \in X$. Thus $\mathbf{v} \in X^\perp$.