- PRINT NAME _____
- Write version on your blue book and hand in this exam inside your blue book.

VERSION B

- There are a total of 50 points possible.
- ONE PAGE of notes is allowed. No calculators are allowed.
- **You must show your work to receive credit.**

1. (6 pts) Find a matrix T so that, if $\mathbf{x} \in \mathbb{R}^3$ has coordinates $\mathbf{c}$ in the basis

$$\mathbf{v_1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \mathbf{v_2} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad \mathbf{v_3} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix},$$

then it has coordinates $T\mathbf{c}$ in the standard basis $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ for $\mathbb{R}^3$.

2. (24 pts) A linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ is given by $L(\mathbf{x}) = \begin{pmatrix} x_1 + x_2 \\ x_2 + x_3 \\ x_1 - x_3 \end{pmatrix}$.
   (a) What is the kernel of L?
   (b) Find a set of vectors that span the range of L. (They need not be a basis.)
   (c) Find a matrix A such that $L(\mathbf{x}) = A\mathbf{x}$.
   (d) What is the dimension of the range of L? Give a reason for your answer.

3. (10 pts) Suppose that $A, B \in \mathbb{R}^{n \times n}$ are nonsingular and that A and B are similar. Prove that A^{-1} and B^{-1} are similar.

4. (10 pts) Suppose V is a subspace of $\mathbb{R}^n$ and W is a subspace of V. Prove that $W^\perp$ contains $V^\perp$.

WARNING: The final exam will probably not be in this room.

END OF EXAM