The two versions are nearly the same and problems 1 and 3 have been interchanged. The solutions here are for version A with notes on changes for B.

1. (#3 in version B) (a) $AB = BA = I$. I mentioned in class that $AB = I$ or $BA = I$ is sufficient, so either "$AB = I$" and "$BA = I$" are also acceptable.

   (b) transpose

2. There are many ways to convert a matrix to row echelon form. I'll choose one way. Rn means row n. For version A:

$$\left(\begin{array}{cccc|cc} 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 & 0 \\ 2 & 2 & 0 & 3 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cccc|cc} 1 & 0 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 1 \end{array}\right) \quad \begin{array}{c}\text{add } (-2)\times(\text{R3}) \text{ to R4} \\ \text{then switch R2 and R3}\end{array}$$

$$\to \left(\begin{array}{cccc|cc} 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right) \quad \begin{array}{c}\text{subtract R1 from R2} \\ \text{add } (-5)\times(\text{R3}) \text{ to R4}\end{array}$$

   Thus (a) has solutions; e.g., $(1\ -1\ 0\ 0)^T$; however (b) does not have a solution because the last row of the row echelon form augmented matrix is inconsistent.

   For version B:

$$\left(\begin{array}{cccc|cc} 1 & 1 & -1 & 0 & 0 & 0 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{cccc|cc} 1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right) \quad \begin{array}{c}\text{add } (-2)\times(\text{R1}) \text{ to R2} \\ \text{subtract R1 from R3}\end{array}$$

$$\to \left(\begin{array}{cccc|cc} 1 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right) \quad \begin{array}{c}\text{move R2 to end, then} \\ \text{add } (-5)\times(\text{R3}) \text{ to R4}\end{array}$$

   Thus (a) has no solutions because the last row of the row echelon form augmented matrix is inconsistent; however, (a) has solutions; e.g., $(0\ 0\ 0\ -1)^T$.

   (c) $A\mathbf{x} = \mathbf{b}$ has either no solutions or an infinite number because there is a free variable (x_3 for A and x_4 for B).

3. (#1 in version B) (a) False. Example: $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$.

   (b) In version A, $(AA^T)_{11} = a_{11}^2 + a_{12}^2 + \cdots + a_{1n}^2 \geq 0$ since it is a sum of squares. In version B, $(A^TA)_{11} = a_{11}^2 + a_{21}^2 + \cdots + a_{n1}^2 \geq 0$.

4. As noted in class, we need only verify either $AB = I$ or $BA = I$ to show that $B = A^{-1}$.
   In version A, $(I - A)(I + A + A^2) = I + A + A^2 - A - A^2 - A^3 = I - A^3 = I$.
   In version B, $(I + A)(I - A + A^2) = I - A + A^2 + A - A^2 + A^3 = I + A^3 = I$.

5. $\det(LU) = \det(L)\det(U)$. The determinant of a triangular matrix is the product of its diagonal entries. Thus $\det(L) = 1$ and $\det(U) = u_{11} \cdots u_{nn}$.