Math 20F First Midterm January 27, 2003

- PRINT NAME ______________________________
- Write version on your blue book and hand in this exam inside your blue book. **VERSION A**
- There are a total of 40 points possible.
- No BOOKS, NOTES or CALCULATORS are allowed.
- **You must show your work to receive credit.**

1. (4 pts.) Fill in the following blanks.

   (a) If A is an $n \times n$ matrix and there is a matrix B such that ____, we call B the inverse of A.

   (b) If A and B are matrices such that $a_{ij} = b_{ji}$ for all i and j, we call B ____.

2. (15 pts.) Let $A = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & -1 \\ 2 & 2 & 0 & 3 \end{pmatrix}$.

   (a) How many solutions does $A\mathbf{x} = (1\ 0\ 0\ 0)^T$ have? Justify your answer.

   (b) How many solutions does $A\mathbf{x} = (0\ 0\ 0\ 1)^T$ have? Justify your answer.

   (c) Either find a $\mathbf{b}$ so that $A\mathbf{x} = \mathbf{b}$ has exactly one solution or explain why this is impossible.

3. (10 pts.) Prove or give a counterexample:

   (a) If A and B are 2×2 matrices, then $AB = BA$.

   (b) For every matrix matrix A, the first entry in AA^T is non-negative. (In other words, if $B = AA^T$, then $b_{11} \geq 0$.)

4. (6 pts) Suppose A is an $n \times n$ matrix and A^3 is the matrix of all zeroes. Prove that $(I - A)^{-1} = I + A + A^2$.

5. (5 pts.) Suppose $A = LU$ is an LU-decomposition of the $n \times n$ matrix A. Recall that in an LU-decomposition L is a lower-triangular with ones on its diagonal and U is an upper triangular matrix. Prove that $\det A = u_{11}u_{22} \cdots u_{nn}$, the product of the diagonal entries of U.

END OF EXAM