1. There may be other answers than these
   (a) gradient, scalar OR gradient, potential
   (b) 0
   (c) harmonic
   (d) gradient, curl OR irrotational function, solenoidal function OR gradient of a scalar potential, curl of a vector potential

2. First solution: Use the divergence theorem and $\nabla \cdot \nabla \times \mathbf{F} = 0$.

   Second solution: Use Stokes' Theorem. Since we are integrating over a closed surface, there is no boundary and so the integral is zero.

3. Since $\mathbf{F}$ is defined everywhere, we can take the domain to be all of 3-space and $\mathbf{R}_0 = \mathbf{0}$.
   (a) Since $\mathbf{F}$ is homogenous of degree 2, the supplementary homework tells us that
   $$\mathbf{G} \;=\; \frac{12}{2+2}(2xz\mathbf{i} - z^2\mathbf{k}) \times \mathbf{R} \;=\; 3(yz^2\mathbf{i} - 3xz^2\mathbf{j} + 2xyz\mathbf{k}).$$
   Another way to solve it is to compute the integral in the text:
   $$\mathbf{G} \;=\; \int_0^1 t(24xzt^2\mathbf{i} - 12z^2t^2\mathbf{k}) \times \mathbf{R}\,dt \;=\; \int_0^1 t(12yz^2t^2\mathbf{i} - 36xz^2t^2\mathbf{j} + 24xyzt^2\mathbf{k})\,dt$$
   $$=\; 3yz^2\mathbf{i} - 9xz^2\mathbf{j} + 6xyz\mathbf{k}.$$
   (b) We can add the gradient of any function to $\mathbf{G}$ without changing its curl. We need ϕ such that $(\mathbf{G}+\nabla\phi)\cdot\mathbf{k} = 0$. The general solution to this is $\phi = -3xyz^2 + f(x,y)$. For simplicity, I took $f = 0$, but you could make another choice. This gave me $\mathbf{G} + \nabla\phi = -12xz^2\mathbf{j}$.

4. Looking at the given equations for D, we see that, in terms of u and v, it is the square
   $$R \;=\; \{(u,v) \mid -1 \le u \le 1 \text{ and } -1 \le v \le 1\}.$$
   We have
   $$\frac{\partial(u,v)}{\partial(x,y)} \;=\; \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \;=\; -2.$$
   Since $\frac{\partial(x,y)}{\partial(u,v)}\frac{\partial(x,y)}{\partial(u,v)} = 1$, we have $\partial(x,y)/\partial(u,v) = 1/-2 = -1/2$. Alternatively, you could express x, y in terms of u, v and compute $\partial(x,y)/\partial(u,v)$ directly. The answer can be written in various ways:
   $$\iint_R v^2e^{uv}\left|\tfrac{-1}{2}\right|\,dA \;=\; \tfrac{1}{2}\iint_R v^2e^{uv}\,du\,dv \;=\; \tfrac{1}{2}\int_{-1}^1\int_{-1}^1 v^2e^{uv}\,du\,dv.$$
   Aside: to evaluate the integral, use $\int te^{at}dt \;=\; \frac{1}{a}te^{at} - \frac{1}{a^2}e^{at} + C$ and write
   $$2\int_{-1}^1\int_{-1}^1 v^2e^{uv}\,du\,dv \;=\; 2\int_{-1}^1 ve^{uv}\Big|_{u=-1}^{u=1}dv \;=\; 2\int_{-1}^1 (ve^v - ve^{-v})\,dv$$
   $$=\; 2(ve^v - e^v + ve^{-v} + e^{-v})\Big|_{v=-1}^{v=1} \;=\; 2(e^{-1}+e^{-1}) - 2(-e^{-1}-e^{-1}) \;=\; 8/e.$$